Communicating Bias*

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Abstract

We consider a static cheap talk model with either one or two senders whose biases are privately known to themselves only. Before the senders learn the state, they send a cheap talk message about their bias to the receiver. Subsequently, the receiver chooses one sender to get state-relevant advice from. We ask two questions - One, is there an equilibrium where the senders' bias is revealed? Two, is the bias revealing equilibrium welfare improving for the receiver? We find that when there is only one sender, there is no bias revealing equilibrium. However, when there are two senders, there exists a bias revealing equilibrium, and it could give the receiver more utility than any equilibrium which is possible without any bias revelation. This highlights a new channel through which sender competition can benefit the receiver.

JEL codes: D82, D83

Keywords - Cheap talk, uncertain bias, multiple senders, bias revelation

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1 Introduction

Consider a cheap talk model of strategic communication in which the receiver is uncertain about the sender's bias. The receiver can interact with multiple experts before hiring one expert from whom she can get payoff-relevant advice. For example, consider an individual who wants to consult a financial adviser. The individual may be uncertain about the risk appetites of different advisers. However, he can talk with different advisers (without disclosing the state of the world - his personal finances) before deciding who he will consult. In this cheap talk interaction, the individual hopes to learn something about the adviser's bias. Subsequently, the individual can hire one adviser and disclose his finances, at which point the adviser learns the state and can give advice via a cheap talk message. There are other examples of such environments like, for legal advice a defendant has to hire one from a pool of lawyers, for policy advice companies have to hire one of several field experts.

Motivated by such examples, we consider a cheap talk game with $N \in \{1,2\}$ senders, where the bias of each sender is her private knowledge. It is common knowledge that the bias can be either *high* or *low* and that the bias of each sender is drawn independently from a known distribution. Before any player (sender or receiver) gets information about the payoff relevant state, we add a bias communication stage (stage 1). In this stage, the senders simultaneously send cheap talk messages about their bias type, following which the receiver selects one sender. The next stage (stage 2) is a cheap talk game in which the chosen sender perfectly observes the true state and sends a cheap talk message to the receiver. The receiver chooses an action, and all players get paid.

First, we investigate whether there is an equilibrium in which the sender(s) reveal their bias in stage 1. Our main result shows that an endogenous bias revelation equilibrium exists when there are two senders, but not for the one sender case. Moreover, under some conditions, the receiver prefers a two-sender bias-revelation equilibrium over any equilibrium possible without the bias revelation stage. Thus, we demonstrate a new channel through which sender competition can benefit the receiver - competition permits an endogenous bias revealing equilibrium which improves the ex-ante payoff of the receiver in the cheap talk game. We also contribute to the literature on cheap talk with uncertain biases by characterizing closed form expressions for equilibria and equilibrium payoffs in a cheap talk game with quadratic loss utility functions.

In the baseline case of one sender, any bias revealing equilibrium will feature a Crawford-Sobel type partition equilibrium (Crawford and Sobel (1982)) in the second stage after biases are revealed in the first stage. For any such equilibria played in stage 2, we show that the gain from deviation for the high-bias sender always exceeds the gain from not deviating for the low-bias sender. This is because the high bias sender obtains a more balanced[1] partition when she pretends to be low-type whereas it is the reverse for the low type sender. Since senders have concave utility functions, they prefer more balanced partitions. This result implies that whenever the high bias sender does not want to deviate (gain from deviation is negative), the low bias sender will want to deviate (gain from not deviating is negative), and thus we cannot have both types revealing their bias in equilibrium.

However, when two senders compete to be hired by the receiver, there exists a bias revealing equilibrium. The two-sender case presents two new forces. First, since the receiver hires only one sender, the senders *compete* to get hired since the sender who is not chosen gets a bad outside option. Second, as the probability of getting hired depends on every sender's bias message, and the senders are uncertain about their rival's bias, there is strategic uncertainty, which is not present in the one-sender case. These new forces allow us to control incentives by identifying conditions on the outside option and on the strategic uncertainty which allow the bias revealing incentive compatibility constraints to hold.

To show the welfare implications of sender competition, we analyze when a bias revealing equilibrium is preferred by the receiver over any equilibrium possible without the bias revealing stage. There are two factors which affect the receiver's payoff which help us compare bias revealing equilibria to those possible without the bias revelation stage. One, the amount of information transmitted as measured by the number of partitions possible in equilibrium. Two, the variance in payoff induced by the equilibrium as measured by the balance[2] of the partition (more balance leads to lower variance in payoff). Since we work with the quadratic loss utility function, the receiver prefers more information and more balance. The maximum number of partitions possible in a non-revelation equilibrium is between the maximum number of partitions possible with a known high bias sender in stage 2 and the number of partitions possible with a known low bias sender

[1] Li and Madarász (2008) define a partition as more balanced than another if there is a smaller increase in noise as one moves from lower messages to higher ones (for positive biases). Receivers with concave utility functions prefer more balanced partitions.

[2] Given a partition with n intervals, if all intervals are of equal length then the partition is most balanced whereas if the variance in interval sizes is very high, the partition has low balance.

in stage 2[3]. Thus, compared to a non-revealing equilibrium, in a revelation equilibrium, the receiver gains/loses in amount of information when stage 2 is the maximal partition equilibrium with low/high bias sender. Further, the variance in payoff induced by the revealing equilibrium is higher when the difference between high and low bias is higher and the fraction of high bias senders in society is neither too high or too low.

We find conditions under which a bias revealing equilibrium is preferred by the receiver in the following manner. We construct a bias revealing equilibrium where the sender and receiver always play the most informative Crawford-Sobel type partition equilibrium after the bias is revealed. Subsequently, we show that if the high bias is sufficiently large and the maximum partitions possible without bias revelation is strictly lower than that possible in a Crawford Sobel type partition equilibrium with a low bias sender[4], then this bias revealing equilibrium is preferred by the decision maker to all equilibrium possible without any bias revelation. The intuition for this result is that as compared to any equilibrium without bias revelation, in the revelation equilibrium the receiver gains when she hires the low bias sender and loses when she hires the high bias sender. The gains come from higher precision, and the losses come from lower precision and lower balance. As the high bias increases beyond $\frac{1}{12}$[5] the non-revelation world is worse for the receiver as both precision and balance falls whereas the revelation equilibrium is made worse only because of a loss of balance. Thus, the payoff from our revelation equilibrium falls slower than the payoff from the non-revelation equilibrium. When the high bias is large enough, the revelation equilibrium is preferred by the receiver.

We contribute to the literature on cheap talk games with uncertain sender bias. The paper closest to ours is Li and Madarász (2008) which discusses a static cheap talk game with one sender of unknown bias. They compare two regimes - one where the sender must announce her true bias before communicating about the state, and another where the sender has no possibility of revealing any information about her bias before sending state relevant information to the receiver. They find that if the utility function of the receiver is concave enough, then he may prefer the regime where the sender's bias is not revealed. Our paper differs from Li and Madarász (2008) in two ways.

[3]As the parameter capturing the fraction of high bias senders in society goes from zero to one, the maximum number of partitions spans the range.

[4]This is characterized by a high enough probability of the sender being the high bias type.

[5]If $\frac{1}{4} > High\ bias \geq \frac{1}{12}$, then the most informative equilibrium is a two partition equilibrium.

One, we allow the senders to choose if they want to reveal their biases by adding a pre-play bias communication stage before any sender gets to observe the true state. This is different from the exogenous bias revelation regime considered in Li and Madarász (2008) for two reasons. First, since a sender's bias is her private information, in most environments, it would be very difficult to explain how an exogenous truthful bias revelation can be enforced. This assumption becomes even more difficult to justify because we show that in the one sender case, endogenous bias revelation is not possible. Furthermore, while Li and Madarász (2008) consider equilibria that follow the exogenous announcement of true bias, they do not consider the conditions needed for the sender to endogenously reveal her bias prior to learning the state. These conditions limit the possible outcomes in equilibrium. Second, we allow for more than one sender. We show that while endogenous bias revelation is not possible with one sender, bias-revealing equilibria exist in the two sender world, and they can even be welfare improving for the receiver.

Quement (2016) considers a model with unknown bias in which there are two senders and the receiver gets messages from both senders sequentially. In our model, the receiver can only get message from one sender, and the sender has the option to reveal her bias before learning state-relevant information. Further, in contrast to Quement (2016), where an increase in the number of senders reduces the receiver's payoff, we find that sender competition can improve the welfare of the receiver. Other papers with uncertain sender bias include Antić and Persico (2020) in which the bias is endogenously determined, and Atakan et al. (2020) which looks at a repeated game environment to determine the optimal path of the stakes involved in the relationship between a sender and a receiver.

The second strand of literature we connect to is the one on cheap talk models with multiple senders. Li (2010) considers a model with multiple senders and privately known bias with different learning protocols. This paper finds that competition improves welfare. In contrast, we introduce sender competition in a different way: only one sender is hired after the pre-play bias communication stage, so the senders compete to get hired, without knowing the state. Li et al. (2016) considers a model of cheap talk with multiple senders, where each sender gets a private signal about their own project. In contrast, in our model, there is only one payoff relevant state; and once hired, the sender learns the state perfectly[6].

[6]Schmidbauer (2017) considers a multi-period version of Li et al. (2016) and shows more competition harms the

Our paper also relates to multi dimensional cheap talk literature, since there are two dimensions (sender bias and state of the world) which are privately observed by the sender. Multi-dimensionality of the state of the world in itself generates an additional benefit of sender competition, as documented in Battaglini (2002). In our paper, we find a different channel through which competition helps - by allowing for bias revealing equilibrium to exist. Chakraborty and Harbaugh (2007), Chakraborty and Harbaugh (2010) consider multi-dimenisonal cheap talk environments and show how using comparative ranking we can get full information revelation. In our paper, the two dimensions are independent of each other and hence can not be comparatively ranked. Our focus is not on full information revelation across both dimensions, we are interested in examining the bias-revealing equilibria and their welfare properties.

Finally, we contribute to the growing literature on papers which use mechanism design in cheap talk games. We allow the receiver to design a mechanism (without transfers) where she can only reward bias announcements with an equilibrium choice in stage 2 to elicit the true bias of the sender. This is different from mechanism design with transfers as has been studied before by Krishna and Morgan (2008) and Ottaviani (2000) where the receiver can commit to message-contingent transfers to make the sender reveal the state truthfully. Further, in our paper, the message on which the transfer is based is about the senders' bias as opposed to the state-relevant message. For the same reasons, and because we have a static model, we differ from papers like Kolotilin and Li (2021) which looks at a repeated communication framework with transfers, and papers which study relational contracts in a repeated communication framework like Golosov et al. (2014) and Kuvalekar et al. (2022).

The rest of the paper is organized as follows: section 2 describes the model. In section 3, we start with a baseline case of one sender and show that there does not exist a bias revealing equilibrium in the one sender model (subsection 3.1). Subsequently, in section 3.2 we show that such an equilibrium does exist in the two-sender model. Next, we show that a bias-revealing equilibrium may give the receiver more utility than any equilibrium possible without bias revelation. Section 4 discusses some of our modelling choices and concludes the paper.

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information transmission and reduces payoffs.

2 Model

We consider a static strategic communication game with one receiver (he) and $N \in \{1,2\}$ senders (she). It is common knowledge that the state of the world, θ, is uniformly distributed on the unit interval $[0,1]$. The receiver is required to hire exactly one sender, who then learns the state perfectly and sends a cheap talk message to the receiver. Subsequently, the receiver takes an action.

If the true state is θ, the hired sender is sender i, and the receiver takes the action y, then the payoffs are as follows:

$$\begin{aligned} U_R(\theta,y) &= -(y-\theta)^2 \\ U_i(\theta,y,b_i) &= -(y-\theta-b_i)^2 \\ U_{j\neq i} &= -A_{b_j} \end{aligned}$$

where b_i is the bias of sender i, and the sender who is not selected (for $N=2$) gets a reservation payoff of $-A_{b_j}$ for $b_j \in \{b_l, b_h\}$. An sender j's bias b_j is her private information, but it is common knowledge that biases are drawn IID from the distribution:

$$b_i = \begin{cases} b_h & \textit{with probability } p \in (0,1) \\ b_l & \textit{with probability } 1-p \end{cases}$$

where $0 < b_l < b_h$. We assume that the biases are low enough: $b_i < \frac{1}{4}\ \forall\ i$[7]. Further, let $A_l = A$, $A_h = A + c$, where $c \geq 0$ captures the idea that the high bias sender's outside option is allowed to be worse than the low bias sender's outside option. To make sure that senders always want to get hired, we assume their reservation payoff is weakly worse than the lowest possible equilibrium payoff obtained by any sender from being hired, that is, worse than the payoff from a babbling equilibrium in the Crawford-Sobel world.

Two stage game

We consider a two-stage game in which, in the first stage, the players play a cheap talk game that could reveal information about the type of the sender, i.e., her bias.[8] Following this, the receiver

[7] This ensures that there exists at least one informative cheap talk equilibrium when the hired sender's bias is known.

[8] Note that an sender learns the state only after being hired. Therefore, in stage 1, the only information that can be conveyed is about the senders' bias.

decides to hire an sender, who then learns the state perfectly. In the second stage, the hired sender can send a cheap talk message to the receiver about the state of the world. Subsequently, the receiver takes an action.

To contrast our results with those of the non-disclosure world in Li and Madarász (2008), we will also consider a strategic communication game where the senders do not have the option to reveal their types. This game will not have the first stage. We will refer to this game as the *LM* game, and its equilibria as *LM* equilibria. The analysis for such a game is equivalent to the non-disclosure environment analysis presented in Li and Madarász (2008). Next, we explain the timing of the game, and then the notion of Perfect Bayesian Equilibrium in our context. The formal definition of PBE is given in the appendix.

The timing of the game is as follows. At the beginning of the game in stage 1, each sender j privately learns her own type (bias) $b_j \in \{b_h, b_l\}$ and then simultaneously sends a costless message $\mu_j(b_j) \in \mathscr{M}_b$ to the receiver that potentially conveys information about their own bias. Without loss of generality, we focus on direct mechanisms, so, $\mathscr{M}_b = \{b_h, b_l\}$. The receiver then chooses to hire one sender according to a hiring rule $h(\mu_1(b_1), \mu_2(b_2))$ that depends on the observed message vector sent by both senders, and $h_j(\mu_1(b_1), \mu_2(b_2))$ denotes the probability of hiring sender j. Note that if $N = 1$, the receiver is required to hire the sender with probability one.

In stage 2, the hired sender i (with bias b_i) learns the true state θ perfectly and sends another message $\mu_{ib_i}(\theta) \in \mathscr{M}$ to the receiver possibly conveying some information about the state. Focusing on direct mechanisms, we assume $\mathscr{M} = [0,1]$. Upon observing $\mu_{ib_i}(\theta)$, the receiver updates his belief about true state θ and takes an action $y(\mu_{ib_i}(\theta)) \in [0,1]$. Note that stage 2 is exactly like Crawford and Sobel (1982) if the bias of the chosen sender is fully revealed in stage 1. If not, then stage 2 looks like the non-disclosure world of Li and Madarász (2008).

A Perfect Bayesian equilibrium consists of a profile of strategies for the receiver and all senders, and belief vectors such that: (a) given the strategies of all players, the beliefs are derived using Bayes' rule whenever possible; (b) the receiver's hiring rule h and the action function y, maximize his ex-ante expected utility given his belief and the equilibrium strategy of all senders; and (c) for each type of sender, their messaging strategy should maximize their expected payoff given the strategies of all the other senders and the receiver.

Notation

For ease of exposition, we denote $CS_{b_{j'}}^{b_j}(k)$ as the payoff for a b_j type sender in a k partition equilibrium when the sender's bias is thought to be $b_{j'}$ with probability 1 $(b_j, b_j' \in \{b_h, b_l\})$. Thus, when senders are truth-telling, from Crawford and Sobel (1982), we get:

$$CS_{b_h}^{b_h}(m) = -\frac{1}{12m^2} - \frac{b_h^2(m^2+2)}{3}; CS_{b_l}^{b_l}(n) = -\frac{1}{12n^2} - \frac{b_l^2(n^2+2)}{3}$$

In the appendix, we derive expressions for the payoffs $CS_{b_l}^{b_h}(n)$ and $CS_{b_h}^{b_l}(m)$ and how they impact equilibrium. Let an n partition cheap talk equilibrium when the bias of the sender is known to be b_j be denoted by $E_{CS}(b_j, n)$. Let N_{b_j} denote the maximum number of partitions possible in such an equilibrium.

3 Analysis

We start with our benchmark case of one sender. Is truthful bias revelation possible in equilibrium? We show that it is not possible. We show later that a bias revealing equilibrium exists in the two-sender world. Moreover, under some conditions, the bias-revealing equilibrium can give the receiver a higher utility than any equilibrium possible without the bias-revelation stage (stage 1). Thus, this analysis demonstrates a new channel through which competition between senders can improve the welfare of the receiver.

3.1 One sender world

Our main result in this subsection is that with one sender, there does not exist an informative bias revealing equilibrium[9]. This is not intuitive at first glance. In stage 2 of the game, the receiver can promise different cheap talk partition equilibria as a reward to the sender for revealing her bias in stage 1. For example, the receiver may compensate the higher bias sender with a finer partition

[9]Note that it is obvious that if the receiver plays a babbling equilibrium in stage 2 irrespective of the messages received in stage 1, then both types of senders will have no incentive to deviate from truth telling in the bias revealing stage. However, since this equilibrium is uninteresting (though the bias is revealed in equilibrium, this is not payoff relevant for the receiver), we will only consider those equilibria in stage 2 where the receiver is playing a non-babbling cheap talk equilibrium after at least one of the stage 1 messages.

equilibria[10] for revealing her bias as compared to the equilibrium after revealing the lower bias.[11] However, we show that no matter which equilibria are played after stage 1, either the low bias sender or the high bias sender (or both) will want to deviate from a bias revealing strategy in stage 1. This is in contrast to the two sender case (section 3.2) where we show that there exist equilibria where the senders truthfully reveal their bias in equilibrium. Proposition 1 shows our main result for this section.

Proposition 1. *When there is only one sender, there is no bias revealing equilibrium in pure strategies.*

Proof. The proof is presented in the appendix. □

We give a short explanation for the non-existence of a bias revealing equilibrium in the one sender case. Suppose that there is a bias revealing equilibrium where the high bias message results in $E_{CS}(b_h, m)$ equilibrium (that is, an m partition CS b_h equilibrium) and the low bias message results in $E_{CS}(b_l, n)$ equilibrium. If $m \leq n$, it is easy to show that the high bias sender would deviate and pretend to be low bias since she will be able to obtain higher actions in equilibrium, whereas the low bias sender would want to announce her type honestly. That is, the gain from deviation is non-negative (i.e. $CS_{b_l}^{b_h}(n) - CS_{b_h}^{b_h}(m) \geq 0$) for the high bias sender and the gains from not deviating is non-negative (i.e. $CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m) \geq 0$) for the low bias sender. Now consider any $m > n$ and the difference between these two differences i.e.:

$$[CS_{b_l}^{b_h}(n) - CS_{b_h}^{b_h}(m)] - [CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)]$$
$$= (b_h - b_l)^2(2 - \frac{1}{n} - \frac{1}{m}) + \frac{(b_h^2 - b_l^2)}{3}(m^2 - n^2) \tag{1}$$

This expression is clearly positive since $m > n$, $b_h > b_l$ and at least one of m, n is greater than 1 (else it would be a babbling equilibrium and we have stated before - we are interested in only informative equilibria). Thus, as we increase m to incentivize the high bias sender to not deviate, before the gains from deviating become negative for the high bias sender (i.e. high bias sender wants to reveal her

[10] To the extent allowed by the size of the bias.

[11] The reader may wonder if the lower bias sender will want to deviate and lie about her type to benefit from the finer partition offered to the higher bias sender in such an equilibrium. The lower bias sender's incentive compatibility constraint will be satisfied if the partition points of the high bias sender are unbalanced (many small partitions at the lower levels of the state with larger partitions at the higher levels) enough. This is because of the risk averse utility function of the senders.

bias), the gains from not deviating become negative for the low bias sender (i.e. the low bias sender wants to lie and announce her type to be high bias). Therefore, we cannot incentivize both types of senders to reveal their bias truthfully in any equilibrium. The intuition for the higher deviation gains for the high bias sender is that the high bias sender obtains a more balanced partition when she lies and pretends to be a low bias sender, whereas it is the other way around for the low bias sender.

So, what kind of equilibria exist when there is only one sender? From our result, we know that no equilibrium perfectly reveals the sender's bias, and thus any equilibria will feature uncertainty about the sender's bias in stage 2. In these environments, 'conflict hiding' (see Li and Madarász (2008)) equilibria may exist[12]. Example 1 in appendix B demonstrates one such equilibrium.

3.2 Two sender world

Next, we consider the environment with two senders. In stage 1 (bias revealing stage), the senders do not know the state, but they simultaneously send cheap talk messages to the receiver (about their bias). Subsequently, the receiver hires one of them. In stage 2, the hired sender gets to see the true state and sends a cheap talk message about the same. The sender who was not hired receives an outside option $-A_{b_j}$ where b_j is the bias of that sender. We assume that $-A_{b_j} < CS_{b_j}^{b_j}(1)$ i.e. not being hired guarantees a payoff worse than the lowest equilibrium payoff if hired (from a babbling equilibrium). This makes sure that all senders prefer getting hired in equilibrium.

3.2.1 General bias revealing equilibrium

In any equilibrium of the two-stage game, if the bias is revealed in the first stage, only partition equilibrium a la Crawford and Sobel (1982) is possible in the second stage.

Since our objective is not just to find a bias revealing equilibrium, but also to find conditions under which it gives the receiver a higher utility than any equilibrium possible without the bias revealing stage, we will focus our attention on only the most informative bias revealing equilibrium i.e. one in which the sender and receiver play the highest partition cheap talk equilibrium possible with the chosen sender in stage 2 of the game. We use the notation $S_{RE,v}$ to refer to the highest partition bias revealing strategies where the b_h type sender is chosen with probability v when the two

[12]Conflict hiding equilibria feature both types of senders sending the same messages in equilibrium (thus the messages do not reveal the type) but over different partitions.

senders send different bias messages. The corresponding bias revealing equilibrium (if it exists) is denoted by $E(RE,v)$. The utility of the decision maker from this equilibrium is given by $U_R(RE,v)$. Below we describe strategy profile $S_{RE,v}$.

Definition 1. *The bias revealing strategy profile* $S_{RE,v} = (\mu_i(b_i), h(b_i,b_j), \mu_j, y)$ *is defined as:*

$$
\begin{aligned}
&\textit{Stage } 1\\
&\mu_i(b_i) = b_i \ \forall \ b_i \in \{b_l, b_h\} \textit{ and } i \in \{1,2\}\\
&h(b_l,b_l) = \left(\frac{1}{2},\frac{1}{2}\right), \quad h(b_h,b_h) = \left(\frac{1}{2},\frac{1}{2}\right)\\
&h(b_l,b_h) = (1-v,v), \quad h(b_h,b_l) = (v,1-v), \quad v \in [0,1]\\
&\textit{Stage } 2\\
&\textit{For any reports where the sender who announced } b_k \textit{ is hired:}\\
&\textit{Play } E_{CS}(b_k,N_{b_k}) \textit{ equilibrium in period 2}\\
&\textit{Deviation by hired sender in stage 2: take the lowest equilibrium action in } E_{CS}(b_k,N_{b_k})\\
&\textit{Deviation by the receiver in hiring in stage 1: Play a babbling equilibrium with the hired sender}
\end{aligned} \tag{2}
$$

This strategy profile requires the senders to truthfully announce their bias in stage one of the game. If both senders announce the same bias b_k then the receiver randomly hires one of them, and the hired sender and receiver play the $E_{CS}(b_k,N_{b_k})$ equilibrium in stage 2. If the senders announce different biases, then with probability v the receiver hires the sender who announces b_h and subsequently they play the $E_{CS}(b_h,N_{b_h})$ equilibrium in stage 2, and with probability $(1-v)$ the receiver hires the sender who announces b_l and subsequently they play the $E_{CS}(b_l,N_{b_l})$ equilibrium in stage 2. We will find parametric conditions under which the bias-revealing strategy profile described in 2 constitutes a Perfect Bayesian Equilibrium. Subsequently, we will find sufficient conditions under which the bias revealing equilibrium is preferred by the decision maker to any equilibrium which is possible without the bias revelation stage. The following proposition shows conditions under which the bias revealing strategy profile can be an equilibrium.

Proposition 2. *For* $v = [0,1)$*, then the bias revealing strategy profile* $S_{RE,v}$ *is not an equilibrium. If* $v = 1$*, given any* $b_l \in (0,1/4)$*, there exists a* p' *and* $\overline{b}$*, such that if* $p < p'$ *and* $b_h \geq \overline{b}$*,* $S_{RE,v=1}$

constitutes an equilibrium $E(RE, v = 1)$.

Proof. The proof is in the appendix. □

We present the intuition for the proof here. A stochastic hiring rule $v \in (0,1)$ can not be part of an equilibrium, because following a mixed message vector announced in stage 1, the receiver will deviate from mixing without being detected and always pick the low bias sender. When $v = 0$, then the receiver is always supposed to pick the low bias sender when the bias announcements in stage 1 are different. In this case, it is not possible to satisfy the incentive compatibility condition of the high bias sender who will always deviate and announce her bias to be low.

When $v = 1$, the high bias sender is always hired following a mixed message vector, this makes it difficult to incentivize the low bias sender to reveal true type. Only when p be low, that is, the probability of facing a high type sender is low for a low bias sender, the risk of not getting hired is reduced. Even with this constraint, the low bias sender may feel that she can increase the probability of her selection if she announces that her type is b_h. However, when b_h becomes sufficiently high, N_{b_h}, that is, the number of partitions offered to the high biased sender after hiring falls, lowering the deviation payoff.

The above equilibrium $E(RE, v = 1)$ requires that ex-post, once the types are revealed, the receiver would choose a suboptimal action, namely choosing the high bias sender b_h [13]. In the next section, we justify the existence of such an equilibrium by exploring the ex-ante welfare implications of this ex-post inefficient choice.

3.2.2 Welfare

In this section, we compare the expected utility of the receiver under the bias revelation equilibrium $E(RE, v = 1)$ and the best equilibria possible without the bias-revealing stage. These equilibria when there is no bias revealing stage (stage 1) are studied as conflict hiding equilibria in Li and Madarász (2008). We refer to them as LM equilibria. A k partition LM equilibrium when the bias of the sender is believed to be b_h with probability p is denoted by $E_{LM}(p,k)$.

Comparing the bias revelation equilibrium $E(RE, v = 1)$, we show that an LM equilibrium $E_{LM}(p,k)$ guarantees higher utility for the receiver whenever the number of partitions in the LM

[13] Whenever this type exists.

equilibrium = $k = N_{b_l}$. If k is strictly less than N_{b_l}, we find that the bias revelation equilibrium is better than LM equilibria only when b_h is sufficiently high.

Lemma 1. *Suppose $E(RE, v = 1)$ exists with the strategy profile $S_{RE,v=1}$. If the number of partitions in the LM equilibrium $E_{LM}(p,k)$ is $k = N_{b_l}$, then the $E_{LM}(p,k)$ is preferred to the revelation equilibrium $E(RE, v = 1)$ by the receiver.*

Proof. The proof is in the appendix. □

Since the receiver's utility function is concave, she prefers to have lower variance in her payoffs. When biases are not revealed, both types of sender use the same number of partitions (albeit with different cutoffs). Further, in this case, the number of partitions in equilibrium is the highest possible amongst the two types of senders. When the biases are revealed, the payoff difference between hiring the two types becomes more extreme (and therefore less desirable), as the high-bias sender can only support a low number of partitions in equilibirum compared to the low bias type. We show that the quadratic utility function is concave enough that the receiver prefers the equilibrium without bias revelation in this case. The next proposition provides a necessary condition for the receiver to prefer the bias-revealing strategy profile to all equilibrium possible without any bias revelation.

Proposition 3. *Suppose the bias revealing equilibrium $E(RE, v = 1)$ exists. This equilibrium is preferred by the receiver to any LM equilibrium $E_{LM}(p,k)$ only if $b_h > 0.204$ and $k < N_{b_l}$.*

Proof. In the appendix. □

The intuition for this result is as follows. When the maximum partitions possible without bias revelation is strictly less than N_{b_l}, as compared to any equilibrium without bias revelation, we know that in the revelation equilibrium the receiver gains when she hires the low bias sender and loses when she hires the high bias sender. The gains come from higher precision (bigger partitions with the low bias sender), and the losses come from lower precision and lower balance (when she hires the high bias sender). As b_h increases beyond $\frac{1}{12}$[14] the non-revelation world is worse for the receiver as both precision and balance falls whereas the revelation equilibrium is made worse only because of a loss of balance. Thus, the payoff from a revelation equilibrium falls slower than the payoff from

[14]If $\frac{1}{4} > b_h \geq \frac{1}{12}$, then $N_{b_h} = 2$

the non-revelation equilibrium. We show that when b_h is high enough the revelation equilibrium is preferred by the receiver.

As we have seen, for a small range of parameters[15], it is possible to have a bias revealing equilibrium which is preferred by the receiver to any equilibrium possible without the bias revealing stage. In the next section, we demonstrate how we can obtain this result for a larger parameter space.

3.3 Public Randomization Device

The bias revealing equilibrium with $v = 1$ requires the sender to always hire the high bias sender whenever the bias messages of the two senders are different. This reduces the maximum payoff possible for the receiver from the bias revealing equilibrium $E(RE, v = 1)$. If we allow for a positive probability of hiring for the low bias type of sender in case of mixed messages, this will improve receiver's payoff. However, as we mentioned before, such a mixed hiring strategy is not incentive compatible for the receiver since after observing a mixed message in Stage 1, she would always prefer to select the low-bias sender instead of actually mixing in equilibrium (if $v \in (0,1)$). The issue with implementing such an equilibrium is that deviation from the mixed strategy by the receiver is not observable, and therefore not punishable.

If we allow the receiver to have access to a public randomization device[16], it can solve the commitment problem and we will be able to get a larger set of mixed strategies as part of the equilibrium. In the one sender world, introducing this public randomization device allows us to expand the possible equilibria in Stage 2 game: now the receiver can commit to mixing between $CS_{b_j}^{b_j}(x)$ and $CS_{b_j}^{b_j}(y)$ as long as $x, y \leq N_{b_h}$, and use this as a threat to incentivize truth-telling in Stage 1[17]. Our first result is that there cannot be a bias revealing equilibrium in the one sender case even after allowing for mixed strategies by the receiver (Proposition 4). In a two sender world, a public randomization device permits us to use mixed hiring strategies in addition to mixed strategies employed in stage 2. Such mixed hiring strategies can be used to improve the receiver's welfare,

[15] b_h can only take values in $(\frac{1}{\sqrt{24}}, \frac{1}{4})$.

[16] Suppose we wish to sustain an equilibrium where the high bias sender is hired with probability v ($\in (0,1)$) when the senders announce different biases in stage 1. Then a public randomization device can be thought of as a biased coin which takes heads with probability v. The coin is tossed and if the outcome (publicly observed) is heads, then the receiver is supposed to pick the high bias sender and if its tails, then the receiver is supposed to pick the low bias sender.

[17] In one sender world, hiring strategies can not be mixed because we have assumed that hiring a sender is always strictly better than not hiring anyone.

and indeed we find that for a broader class of parameters, the revealing equilibrium $E(RE,v)$ exists and generates higher welfare for the sender (Proposition 5 and 6).

Proposition 4. *Suppose that the receiver can commit to mixing in hiring. There does not exist any informative bias-revealing equilibrium with one sender.*

Proof. In the appendix. □

Next, we turn to the two sender case and find a) conditions under which the bias revealing strategy profile in 2 constitutes an equilibrium with $v \in (0,1)$, and b) conditions under which this gives the receiver a higher utility than any equilibrium that can be achieved without the bias-revealing stage 1.

Proposition 5. *There exists a $\overline{v} \in (0,1)$ such that $\forall v \in [\frac{1}{2},\overline{v}]$, there exists a $\overline{c}(v)$, where if $A_{b_h} > A_{b_l} + \overline{c}(v)$, there exists an interval of p in which the bias revealing strategy profile $S_{RE,v}$ constitutes an equilibrium.*

Proof. The detailed proof is in the appendix. Intuitively, when $v < 1$, the incentive compatibility condition for the high bias type becomes harder to satisfy. We solve this problem by lowering her outside option. If p is low enough, then the high bias sender faces the tradeoff of announcing her type truthfully and getting hired with high probability or deviating and obtaining her outside option with probability close to half[18]. If the outside option is very low, then the high bias sender prefers to announce her type truthfully.

□

Proposition 6. *Given any b_h, there exists $\bar{b}$ such that if $b_l < \bar{b}$, there exists an interval of p in which the bias-revealing strategy profile $S_{RE,v}$ constitutes an equilibrium, and in this equilibrium $E(RE,v)$, the receiver enjoys a higher utility compared to any equilibrium $E_{LM}(p,n)$ that can exist without the bias revealing stage.*

Proof. In the appendix. □

[18]When p is low, the other sender is likely to announce her type as b_l. If the high bias sender also announces her type to be b_l, then the receiver tosses a coin and hires one of the senders.

While comparing the bias revealing highest partition equilibrium $E(RE, v)$ to the equilibria possible without the bias revelation stage ($E_{LM}(p, n)$), we identify two key factors that affect the receiver's payoff. One, the amount of information transmitted as measured by the number of partitions possible in equilibrium. Two, the variance in payoff induced by the equilibrium as measured by the balance of the partition (more balance leads to lower variance). Since we work with a quadratic loss utility function, the receiver prefers more information and more balance. The maximum number of partitions possible in a non-revelation equilibrium is n, which lies between the maximum number of partitions possible in stage 2 with the high bias sender (N_h) and the number of partitions possible with the low bias sender (N_l)[19]. Thus, compared to a non-revealing equilibrium $E_{LM}(p, n)$, in a revelation equilibrium $E(RE, v)$, the receiver gains (respectively, loses) the amount of information in stage 2 when the hired sender is low (respectively, high) bias type. Further, the variance in payoff induced by $E(RE, v)$ is increasing in $b_h - b_l$ when p is neither too high nor too low.

We find conditions under which a bias revealing equilibrium is preferred by the receiver. First, with the help of a public randomization device, we construct a bias-revealing equilibrium where the players always play the most informative partition equilibrium in stage 2. Next, we show that fixing the level of high bias, if the lower bias is small enough, then a) the number of partitions possible when communicating with a low bias sender is strictly higher than the number of partitions possible without the bias revelation stage if the fraction of high bias senders is above a cutoff, b) the cutoff needed in the previous point becomes really small. Further, the variance induced by the bias revealing equilibrium is low when the fraction of high bias senders is low. We show that with a large fraction of low bias senders, the receiver's benefit from the extra information (more number of partitions) obtained from the low bias sender in a revealing equilibrium is more than the receiver's loss emanating from low information transmission with the high bias sender and higher variance in payoff[20].

4 Discussion and Conclusion

Here we discuss the assumptions we made, and how they affect the results.

[19] n is decreasing in p, as p varies from zero to one, the maximum number of partitions spans the range.

[20] Compared to an equilibrium without any bias revelation.

4.1 General utility function

In this paper, we assume that both the sender and receiver have quadratic loss function utilities. There are two reasons for this. One, it is a ubiquitous utility function in the cheap talk literature that offers clear interpretations of results. Further, one of our objectives is to show when the bias revealing equilibrium can be better for the receiver compared to equilibria possible without any bias revelation. From Li and Madarász (2008), we know that fixing all other parameters, this result is not possible if the receiver's utility function is too concave. Thus, we work with a fixed but standard utility function. Note that the result from Li and Madarász (2008) does not preclude the possibility that given a fixed concave utility function, we will be able to replicate our results. However, without closed-form solutions to aid us, we are uncertain about tractability of the problem, and how instructive the results will be.

4.2 Renegotiation

The bias revealing equilibrium we obtain in proposition 3 and 5 is ex-ante better for the receiver than all equilibria without any bias revelation. However, we support this with strategies that are ex-post inoptimal. For example, if the receiver is supposed to pick the sender who announced the high bias but instead deviates and picks the sender who announced the low bias, the hired sender and receiver are required to play the babbling equilibrium in period 2 on this off-equilibrium path. As babbling is an equilibrium, deviation is not possible. However, if renegotiations were permitted, then the hired low bias sender and receiver could play the most informative equilibrium in stage 2. Allowing for renegotiations makes it impossible for a bias revealing equilibrium to exist since the receiver will always pick the low bias sender[21]. While the possibility of renegotiation disallow bias revealing equilibrium, they strengthen our one-sender result that endogenous bias revealing is not possible.

4.3 Conclusion

We consider a model of strategic information transmission in which senders privately know their bias and may choose to disclose the same before communicating state-relevant information. We

[21]The result follows from the case of $v = 0$ in proposition 2.

build on the framework developed in Li and Madarász (2008) and make bias-revelation an endogenous choice. We find closed-form expressions for equilibrium payoffs under no bias revelation. Further, we find that when there is only one sender, the disclosure of bias is not possible in equilibrium. With two senders, we identify conditions for bias-revealing equilibria to exist. Moreover, we find that under some conditions the receiver is better off with a bias-revealing equilibrium compared to the best equilibrium possible without bias revelation. This demonstrates a novel indirect channel through which sender competition can benefit the receiver. Unlike Krishna and Morgan (2001) where sender competition improves the receiver's payoff by getting them to reveal the states directly, in our paper sender competition allows for a bias revealing equilibrium which then benefits the receiver via better information in the cheap talk game.

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A General Expression for k-partition Conflict Hiding Equilibrium

Here we derive the expressions for the equilibrium $E_{LM}(p,k)$ which we use for welfare comparison. Notation inconsistency In a k-partition conflict hiding equilibrium, denote the equilibrium partition structure chosen by a b_j type sender to be: $[a_0^{b_j}=0,a_1^{b_j},a_2^{b_j},...,a_{k-1}^{b_j},a_k^{b_j}=1]$. Then, the action chosen by the receiver after receiving the i^{th} message is:

$$y_i = p\frac{a_{i-1}^h+a_i^h}{2}+(1-p)\frac{a_{i-1}^l+a_i^l}{2}$$

Now, since

$$a_1^h=\frac{y_1+y_2}{2}-b_h; a_1^l=\frac{y_1+y_2}{2}-b_l$$

we get

$$y_1=p\frac{\frac{y_1+y_2}{2}-b_h}{2}+(1-p)\frac{\frac{y_1+y_2}{2}-b_l}{2}=\frac{y_1+y_2}{4}-b/2$$

where $b=pb_h+(1-p)b_l$ =expected bias. Similarly, we get for all $2\leq i\leq k-1$:

$$\begin{aligned} y_i &= p\frac{\frac{y_{i-1}+y_i}{2}-b_h+\frac{y_{i+1}+y_i}{2}-b_h}{2}+(1-p)\frac{\frac{y_{i-1}+y_i}{2}+\frac{y_{i+1}+y_i}{2}-2b_l}{2} \\ &= \frac{y_{i-1}+2y_i+y_{i+1}}{4}-b \qquad (3) \\ 2y_i &= y_{i-1}+y_{i+1}-4b \\ (y_{i+1}-y_i) &- (y_i-y_{i-1})=4b \qquad (4) \end{aligned}$$

Plugging this we get,

$$\begin{aligned} y_2-y_1 &= 2y_1+2b \\ y_i-y_{i-1} &= y_{i-1}-y_{i-2}+4b=2y_1+2b+(i-2)4b \\ y_k-y_{k-1} &= 2y_1+2b+(n-2)4b \qquad (5) \end{aligned}$$

This implies

$$y_2 = y_1 + 2y_1 + 2b = 3y_1 + 2b$$
$$y_i = (2i-1)\,y_1 + 2\,(i-1)^2\,b$$
$$y_k = (2k-1)\,y_1 + 2\,(k-1)^2\,b$$

Also, using:

$$\begin{aligned} y_k &= \frac{p}{2}\left(\frac{y_{k-1}+y_k}{2}+1-b_h\right)+\frac{1-p}{2}\left(\frac{y_{k-1}+y_k}{2}+1-b_l\right) \\ &= \frac{y_{k-1}+y_k}{4}-\frac{1}{2}-\frac{b}{2} \\ y_k &= 1-y_1-2\,(k-1)\,b \end{aligned} \tag{6}$$

Equating expressions for y_k, we solve for the equilibrium actions

$$\begin{aligned} (2k-1)\,y_1+2\,(k-1)^2\,b &= 1-y_1-2\,(k-1)\,b \\ y_1 &= \frac{1}{2k}-b\,(k-1) \end{aligned}$$

For this to be valid, we need:

$$y_1 \geq 0 \Longleftrightarrow \frac{1}{2k}-b\,(k-1) \geq 0 \Longleftrightarrow b \leq \frac{1}{2k\,(k-1)} \tag{7}$$

Similarly, we solve for:

$$y_j = (2j-1)\,y_1+2\,(j-1)^2\,b = \frac{(2j-1)}{2k}+b\left[2j^2-(2j-1)\,(k+1)\right] \tag{8}$$

Now, Receiver's payoff from this equilibrium $E_{LM}(p,k)$:

$$U_R(CH,p,k) \;=\; pU_R^h(LM,p,k)+(1-p)\,U_R^l(LM,p,k) \tag{9}$$

where $U_R^b =$ receiver's payoff if the hired sender's type is $b \in \{b_h, b_l\}$.

Calculate U_R^h separately:

$$\begin{aligned} U_R^h &= \int_0^{a_1^h} -(y_1-\theta)^2 d\theta + \int_{a_1^h}^{a_2^h} -(y_2-\theta)^2 d\theta + \cdots + \int_{a_{k-1}^h}^{a_k^h} -(y_k-\theta)^2 d\theta \\ &= \frac{1}{3}\left[\left[(y_1-\theta)^3\right]_0^{a_1^h} + \left[(y_2-\theta)^3\right]_{a_1^h}^{a_2^h} + .. + \left[(y_k-\theta)^3\right]_{a_{k-1}^h}^{1}\right] \\ &= \frac{1}{3}\left[-(1-y_k)^3 - y_1^3 + \sum_{j=1}^{k-1}\left(\left(y_j-a_j^h\right)^3 - \left(y_{j+1}-a_j^h\right)^3\right)\right] \end{aligned} \tag{10}$$

Now, we know that:

$$\begin{aligned} y_j - a_j^h &= -\frac{1}{2k} + b\,[k-2j] + b_h; \\ y_{j+1} - a_j^h &= \frac{1}{2k} + b\,[2j-k] + b_h \end{aligned}$$

So,

$$\left(y_j-a_j^h\right)^3 - \left(y_{j+1}-a_j^h\right)^3 = -2\left(\frac{1}{2k} + b\,[2j-k]\right)^3 - 6b_h^2\left(\frac{1}{2k} + b\,[2j-k]\right)$$

since $(a-b)^3 - (a+b)^3 = -2b^3 - 6a^2 b$ Thus,

$$\sum_{j=1}^{k-1}\left(\left(y_j-a_j^h\right)^3 - \left(y_{j+1}-a_j^h\right)^3\right) = -\left[\frac{k-1}{4k^3} + b^2(k-1)(k-2) + 3b_h^2\frac{k-1}{k}\right]$$

Using this, we get

$$\begin{aligned} U_R^h &= -\frac{1}{3}\left[\frac{k-1}{4k^3} + b^2(k-1)(k-2) + 3b_h^2\frac{k-1}{k} + \left(\frac{1}{2k} + b(k-1)\right)^3 + \left(\frac{1}{2k} - b(k-1)\right)^3\right] \\ &= -\frac{1}{12k^2} - \frac{b^2}{3}(k-1)\left(k+1-\frac{3}{k}\right) - b_h^2\frac{k-1}{k} \end{aligned} \tag{11}$$

Similarly, we can calculate

$$U_R^l = -\frac{1}{12k^2} - \frac{b^2}{3}(k-1)\left(k+1-\frac{3}{k}\right) - b_l^2\frac{k-1}{k} \tag{12}$$

This gives us the final expression for receiver's payoff in the equilibrium $E_{LM}(p,n)$:

$$\begin{aligned} U_R(LM,p,k) &= pU_R^h + (1-p)\,U_R^l \\ &= -\frac{1}{12k^2} - \frac{b^2}{3}\left(k^2-1\right) - d^2 p\,(1-p)\,\frac{k-1}{k} \end{aligned} \tag{13}$$

Now, the receiver's payoff from the best revealing equilibrium is:

$$U_R(RE,v) = \left(p^2 + 2vp\,(1-p)\right)u_m + \left((1-p)^2 + 2\,(1-v)\,p\,(1-p)\right)u_n \tag{14}$$

where u_m (respectively, u_n) is the CS equilibrium payoff for the receiver when there are m (respectively, n) partitions by a high bias (respectively, low bias) sender:

$$u_m = -\frac{1}{12m^2} - \frac{b_h^2}{3}\left(m^2-1\right); u_n = -\frac{1}{12n^2} - \frac{b_l^2}{3}\left(n^2-1\right)$$

Simplifying, we get the difference between these two payoffs:

$$\begin{aligned} \Delta(p) &\equiv U_R(RE,v)) - U_R(LM,p,k)) \\ &= p^2\left((1-2v)\,(u_m-u_n) + \frac{k^2-1}{3}d^2 - \frac{k-1}{k}d^2\right) + p\left(2v\,(u_m-u_n) + \frac{k^2-1}{3}2b_l d + \frac{k-1}{k}d^2\right) \\ &\quad + \left(u_n + \frac{1}{12k^2} + \frac{k^2-1}{3}b_l^2\right) \end{aligned} \tag{15}$$

We use this expression for the proof of Proposition 6.

B Proofs and Examples

Example 1. *Suppose* $b_l = \frac{1}{6}, b_h = \frac{1}{5}$ *and* $p = \frac{3}{5}$*. The following strategy profile constitutes a perfect Bayesian Equilibrium.*

Stage 1

$\mu_i(b_i) = b_l \ \forall\ i$ *and* $b_i \in \{b_l, b_h\}$

$h(b) = 1\ \forall b \in \{b_l, b_h\}$

Stage 2

Sender Strategy:

$\mu_{ib_l}(\theta) = m_1 \ if\ \theta \in [0, 0.146],\ else\ m_2$

$\mu_{ib_h}(\theta) = m_1 \ if\ \theta \in [0, 0.113],\ else\ m_2$

Receiver strategy:

If receiver observes b_l *in stage 1 and* m_1 *in stage 2, then* $y\left(\mu_{ib_l}(\theta)\right) = 0.0631$

If receiver observes b_l *in stage 1 and* m_2 *in stage 2, then* $y\left(\mu_{ib_l}(\theta)\right) = 0.5631$

If receiver observes b_h *in stage 1, then* $y\left(\mu_{ib_h}(\theta)\right) = 0.0631$

If receiver observes b_i *in stage 1 and* $m \neq m_1, m_2$ *in stage 2, then* $y\left(\mu_{ib_l}(\theta) \notin (m_1, m_2)\right) = 0.0631$

Beliefs: About sender

$p(b_h|m_i) = p\ \forall$ *messages in stage* 1

Beliefs: About state

$$
\begin{aligned}
&P = p(b_h|m_1)U[0, 0.146] + p(b_l|m_1)U[0, 0.113] \text{ ; if receiver observes } b_l \text{ in stage 1 and } m_1 \text{ in stage 2} \\
&P = p(b_h|m_2)U[0.146, 1] + p(b_l|m_2)U[0.113, 1] \text{ ; if receiver observes } b_l \text{ in stage 1 and } m_2 \text{ in stage 2} \\
&P = U[0, 0.146]] \text{ ; if receiver observes } b_h \text{ in stage 1} \\
&P = p(b_h|m_1)U[0, 0.146] + p(b_l|m_1)U[0, 0.113] \text{ ; if receiver observes } b_i \text{ in stage 1 and } m \neq m_1, m_2 \text{ in stage 2}
\end{aligned} \tag{16}
$$

The receiver's expected payoff in this equilibrium[22] *is* -0.056.

[22] $U[a, b]$ represents Uniform distribution with support [a,b]

Proof of proposition 1

Proof. We will prove this result by contradiction. Suppose there exists a bias revealing equilibrium in pure strategies. WLOG, let the equilibrium be the following:

Stage 1:

$\mu_b(x) = x \; \forall \; x \in \{b_l, b_h\}$

$h(\mu_b) = 1 \; \forall \; b \in \{b_l, b_h\}$

Stage 2:

If sender reports type b_l in stage 1: Play an n-partition CS b_l equilibrium

If sender reports type b_h in stage 1: Play an m partition CS b_h equilibrium

If the receiver arrives at an off equilibrium node, she takes the lowest equilibrium action in n partition CS b_l equilibrium

First, let us consider the incentives of the high bias sender. If she plays according to the strategies proposed above, her expected payoff is:

$$\frac{-1}{12m^2} - \frac{{b_h}^2(m^2+2)}{3} \tag{17}$$

Clearly there is no reason to deviate in stage 2 of the game if she reveals her type truthfully in stage 1 (since stage 2 play is an equilibrium, there is no incentive to deviate). If she deviates in stage 1 and reports her type to be b_l, then in stage 2 she can exploit the n partition CS b_l equilibrium to her advantage. In particular, while she does not have the incentives to deviate from the equilibrium messages (else the receiver plays the action $\frac{1}{2}$), she will change the interval of the state space on which the messages are reported (a la Li and Madarasz's conflict hiding equilibrium). In an n partition CS b_l equilibrium, the equilibrium actions are given by

$$y_i = \frac{2i-1}{2n} + b_l(2i^2 + (1+n)(1-2i))$$

where $i = 1, 2, ..., n$. Now, in equilibrium, the high bias sender will not deviate from the messages the low bias sender was meant to send in equilibrium (else the receiver takes the action half). However, the high bias sender does not have to choose the same partition function as the low bias sender. In fact, she will choose cut off points on the state space to maximize her own payoff from the equilibrium messages. In particular, in equilibrium, she will choose points $a_1, ..., a_{n-1}$ such that $a_i + b_h = \frac{y_i + y_{i+1}}{2}$. When the state is between a_i and a_{i+1}, the sender will send the message so that action y_i will be played in response. The expected payoff to the high bias sender from deviating is therefore given by:

$$CS_{b_l}^{b_h}(n) = \int_0^{a_1} -(y_1 - \theta - b_h)^2 \, d\theta + \int_{a_1}^{a_2} -(y_2 - \theta - b_h)^2 \, d\theta + ... \int_{a_{n-1}}^{1} -(y_n - \theta - b_h)^2 \, d\theta$$

Substituting the expressions for y_i and a_i and simplifying, we get that the expected payoff to the high bias sender from deviating is

$$CS_{b_l}^{b_h}(n) = \frac{-1}{12n^2} + {b_l}^2(\frac{4}{3} - \frac{1}{n} - \frac{n^2}{3}) + b_l b_h \frac{2(1-n)}{n} - \frac{{b_h}^2}{n} \tag{18}$$

Comparing 17 and 18, we get that the high bias sender will not deviate if:

$${b_l}^2(4 - \frac{3}{n} - n^2) + {b_h}^2(m^2 + 2 - \frac{3}{n}) - b_l b_h \frac{6(n-1)}{n} \;\leq\; \frac{1}{4}(\frac{1}{n^2} - \frac{1}{m^2}) \tag{19}$$

Inequality 19 captures the incentive compatibility constraint of the high bias sender for the prescribed strategies to constitute an equilibrium.

Now, let us consider the incentives of the low bias sender. Doing the same analysis as before, we can show that the low bias sender will not deviate from the prescribed strategies if:

$$-{b_h}^2(4 - \frac{3}{m} - m^2) - {b_l}^2(n^2 + 2 - \frac{3}{m}) + b_l b_h \frac{6(m-1)}{m} \;\geq\; \frac{1}{4}(\frac{1}{n^2} - \frac{1}{m^2}) \tag{20}$$

Looking at the bias revealing incentives of the two types of senders jointly, we see that 19 and 20 can simultaneously hold only if:

$$-{b_h}^2(4 - \frac{3}{m} - m^2) - {b_l}^2(n^2 + 2 - \frac{3}{m}) + b_l b_h \frac{6(m-1)}{m} \geq {b_l}^2(4 - \frac{3}{n} - n^2) + {b_h}^2(m^2 + 2 - \frac{3}{n}) - b_l b_h \frac{6(n-1)}{n} \tag{21}$$

$$\iff (b_h - b_l)^2(2 - \frac{1}{n} - \frac{1}{m}) \leq 0 \tag{22}$$

This inequality cannot hold unless $b_h = b_l$ or $n = m = 1$ (only babbling equilibrium is played).

Since we have assumed that $b_l < b_h$ and we are only looking for non-trivial equilibria in stage two, we conclude that the proposed strategies do not constitute an equilibrium since at least one type of sender will have incentives to deviate from truth-telling in period 1. □

Proof of Proposition 2

Lemma 2. *For $v = 0$, then the bias revealing strategy profile in 2 is not a Perfect Bayesian equilibrium.*

Proof. Suppose that when the bias is common knowledge, the most informative equilibrium of a b_h, (respectively, b_l) bias sender areceiverits m (respectively, n) partitions. We will show that if $v = 0$, the high bias sender's incentive compatibility constraint is not satisfied.

Under $v = 0$ the IC for b_h type sender is as follows:

$$\underbrace{\frac{1}{2}pCS_{b_h}^{b_h}(m) + \left(1 - \frac{p}{2}\right)(-A_h))}_{\text{payoff from truth-telling}} \geq \underbrace{\frac{1}{2}(1+p)CS_{b_l}^{b_h}(n) + \frac{1}{2}(1-p)(-A_h)}_{\text{payoff from deviating}}$$

Rearranging we get,

$$p(CS_{b_h}^{b_h}(m) - CS_{b_l}^{b_h}(n)) \geq CS_{b_l}^{b_h}(n) + A_h$$

Notice that, RHS= $CS_{b_l}^{b_h} - (-A_h)$= the difference between the high bias sender hiding in an n partition CS b_l equilibrium and a payoff worse than babbling. Clearly, this is positive. Now, we consider the LHS for two cases.

Case 1: $m = n$

We will show that the LHS is negative (at $m = n$) so that the above cannot hold.

$$\begin{aligned} & CS_{b_h}^{b_h}(m) < CS_{b_l}^{b_h}(n) \\ & \Leftrightarrow (b_h - b_l)(1-n)(\frac{b_h - b_l}{n} + \frac{(1+n)(b_h + b_l)}{3}) < 0 \end{aligned}$$

This is true since $n \geq 2$ and $b_h > b_l$.

Case 2: $m < n$

Once again, we will show that $CS_{b_h}^{b_h}(m) < CS_{b_l}^{b_h}(n)$ and thus the high bias sender will deviate and announce her type to be b_l. We will prove this by contradiction. Suppose that $CS^{b_h}_{b_h}(m) \geq CS^{b_h}_{b_l}(n)$. By proposition 1, we know that in the one sender case if the high bias sender does not want to deviate from truth telling then the low bias will want to deviate i.e. $CS^{b_l}_{b_l}(n) < CS^{b_l}_{b_h}(m)$.

Consider $CS^{b_l}_{b_l}(j) < CS^{b_l}_{b_h}(m)$ where j can take any natural number up to n. Fixing b_l, b_h, m, we see that the LHS is increasing in j while the RHS is independent of j. Thus, if we can show that $CS^{b_l}_{b_l}(m) > CS^{b_l}_{b_h}(m)$, then that will imply that $CS^{b_l}_{b_l}(n) > CS^{b_l}_{b_h}(m)$, thereby giving us a contradiction.

$$\begin{aligned} & CS^{b_l}_{b_l}(m) > CS^{b_l}_{b_h}(m) \\ & \Leftrightarrow (b_h - b_l)(m-1)(b_h(\frac{m}{3} - \frac{1}{m}) + b_l(\frac{1}{3} + \frac{1}{m})) > 0 \end{aligned} \tag{23}$$

This is always true for all m>1. □

Lemma 3. *If $v \in (0,1)$, then the bias revealing strategy profile in 2 (denoted by $S_{RE,v}$) is not a Perfect Bayesian equilibrium.*

Proof. In this case, the receiver will deviate when two different biases are announced in stage 1. Since v is strictly between zero and one, 2 requires the receiver to sometimes hire the low bias sender and sometimes the high bias sender, and subsequently play the most informative equilibrium with them. However, since the receiver is always better off with a lower bias sender in the most informative equilibrium, the receiver will deviate from mixing and always choose the low bias sender. □

Lemma 4. *For $v = 1$, given any $b_l \in (0, 1/4)$, there exists a value $\overline{b} > b_l$ and $p_2 > 0$, such that if $b_h \geq \overline{b}$ and $p < p_2$, the bias revealing strategy profile in 2 constitutes an equilibrium.*

Proof. For $v = 1$ let us write the IC for both the b_h and b_l type sender. The IC for b_h is given by,

$$\left(1 - \frac{p}{2}\right) CS^{b_h}_{b_h}(m) + \frac{p}{2}(-A_h) \geq \frac{1}{2}(1-p)CS^{b_h}_{b_l}(n) + \frac{1}{2}(1+p)(-A_h)$$

Rearranging we get,

$$p \geq p_1 \equiv \frac{CS^{b_h}_{b_h}(m) + \frac{1}{2}(A+c) - \frac{1}{2}CS^{b_h}_{b_l}(n)}{\frac{1}{2}(CS^{b_h}_{b_h}(m) - CS^{b_h}_{b_l}(n))} = \frac{(CS^{b_h}_{b_l}(n) - CS^{b_h}_{b_h}(m)) - (CS^{b_h}_{b_h}(m) + (A+c))}{(CS^{b_h}_{b_l}(n) - CS^{b_h}_{b_h}(m))}$$

For this IC to hold we need $p_1 \leq 1$, for $c > 0$ and sufficiently large, this condition always holds true. The IC for b_l is given by,

$$\frac{1}{2}(1-p)CS_{b_l}^{b_l}(n) + \frac{1}{2}(1+p)(-A_l) \geq (1-\frac{p}{2})CS_{b_h}^{b_l}(m) + \frac{p}{2}(-A_l)$$

Rearranging we get,

$$p \leq p_2 \equiv \frac{(\frac{1}{2}CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m) - \frac{1}{2}A_l)}{\frac{1}{2}(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m))} = \frac{(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)) - (CS_{b_h}^{b_l}(m) + A)}{(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m))}$$

This IC can hold only if $p_2 \geq 0$ or

$$CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m) \geq CS_{b_h}^{b_l}(m) + A \tag{24}$$

Note that, given b_l, $CS_{b_h}^{b_l}(m)$ is decreasing in b_h, this is because,

$$\frac{\partial CS_{b_h}^{b_l}(m)}{\partial b_h} = \begin{cases} -2b_h\frac{m^2-4}{3} - \frac{2(b_h-b_l)}{m} - 2b_l & \text{if} \quad \frac{1}{2m(m+1)} \leq b_h < \frac{1}{2m(m-1)} \\ -(2m-1)(\frac{1}{12m^2(m-1)^2} + \frac{b_h^2}{3}) - \frac{(b_h-b_l)^2}{m(m-1)} - 2b_h\frac{m^2-4}{3} - \frac{2(b_h-b_l)}{m} - 2b_l & \text{otherwise} \end{cases}$$

Thus for any $m \geq 2$, $\frac{\partial CS_{b_h}^{b_l}(m)}{\partial b_h} < 0$, which implies as b_h increases the LHS of 24 would increase and the RHS would increase making the IC easier to hold.

Furthemore, as $b_h \to 1/4$, we get,

$$(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)) - (CS_{b_h}^{b_l}(m) + A) = \frac{1}{48} - \frac{1}{12n^2} + \frac{b_l}{2} - \frac{b_l^2(n^2+2)}{3} \geq 0 \quad \forall n \geq 2$$

Thus $p_2 > 0$ as $b_h \to 1/4$. Combining this with the result that as b_h decreases, *LHS* of inequality 24 decreases and *RHS* increases there exists a cutoff value of b_h, namely $\bar{b}$ such that for all $b_h \geq \bar{b}$, $p_2 > 0$. Note that at $b_h = b_l$ since the $LHS = 0$ but $CS_{b_h=b_l}^{b_l}(m=n) > A$, p_2 cannot be positive. This impies $\bar{b} > b_l$.

Thus the IC for b_l is satisfied for $p < p_2$ and $b_h \geq \bar{b}(> b_l)$. □

Proof for lemma 1

Proof. Given equation 7 in appendix A, if the maximum number of partitions of $E_{LM}(p,n)$ is the same as the maximum number of partitions sustained with a b_l sender, that is, N_l, it must be:

$$b \equiv pb_h + (1-p)b_l < \frac{1}{2n(n-1)} \tag{25}$$

$$\Rightarrow p < \frac{\frac{1}{n(n-1)} - b_l}{d}; \tag{26}$$

where $d = b_h - b_l > 0$

The receiver's utility from the revealing equilibrium $E(RE,v)$:

$$U_R(RE,v) = \left[p^2 + 2p(1-p)\right] CS_{b_h}^{b_h}(m) + (1-p)^2 CS_{b_l}^{b_l}(n) \equiv R \tag{27}$$

Whereas the receiver's utility from the n-partition conflict hiding equilibrium $E_{LM}(p,n)$:

$$U_R(LM,p,n) = -\frac{1}{12n^2} - \frac{b^2\left(n^2-1\right)}{3} - \frac{n-1}{n}d^2 p(1-p) \equiv L \tag{28}$$

We need to show that when $n = N_l$, we always have $L > R$.

Rewriting the expressions of 27 and 28 , we get:

$$L = \left[-\frac{(1-p)^2}{12n^2} - (1-p)^2 \frac{b_l\left(n^2-1\right)}{3}\right] + \frac{(1-p)^2}{12n^2} - \frac{1}{12n^2} - p^2 \frac{b_h^2\left(n^2-1\right)}{3}$$
$$-2p(1-p)b_h b_l \frac{\left(n^2-1\right)}{3} - \frac{n-1}{n}d^2 p(1-p)$$

and

$$R = \left[-\frac{(1-p)^2}{12n^2} - (1-p)^2 \frac{b_l\left(n^2-1\right)}{3}\right] + \left[p^2 + 2p(1-p)\right] CS_{b_h}^{b_h}(m)$$

Thus plugging in the value of $CS_{b_h}^{b_h}(m)$, we get $L > R$ would imply,

$$\frac{(1-p)^2}{12n^2} - \frac{1}{12n^2} - p^2\frac{b_h^2(n^2-1)}{3} - 2p(1-p)b_h b_l\frac{(n^2-1)}{3} - \frac{n-1}{n}d^2 p(1-p)$$
$$> \left[p^2 + 2p(1-p)\right]\left(-\frac{1}{12m^2} - b_h^2\frac{m^2-1}{3}\right)$$

Since $n > m$, and $p < 2$, it suffices to show:

$$2p\frac{b_h^2(n^2-1)}{3} - 2p^2\frac{b_h^2(n^2-1)}{3} - 2p(1-p)b_h b_l\frac{n^2-1}{3} - d^2 p(1-p)\frac{n-1}{n} > 0$$
$$\iff 2p(1-p)\left[\frac{b_h(n^2-1)}{3}\right](b_h - b_l) > (b_h - b_l)^2 p(1-p)\frac{n-1}{n}$$
$$\iff 2n\left(\frac{n+1}{3}\right)b_h > (b_h - b_l)$$

Now, we know that

$$b_h > \frac{1}{2n(n-1)} > \frac{1}{2n(n+1)}$$

so,

$$2n\left(\frac{n+1}{3}\right)b_h > \frac{1}{3}$$

Thus, sufficient condition for $L > R$ is:

$$b_h - b_l < \frac{1}{3}$$

which is always satisfied because $b_l < b_h < \frac{1}{4}$ □

Proof of proposition 3

Proof. Suppose the maximum number of partitions sustained in $E(RE, v)$ are m and n, when hiring a high or low bias sender respectively. A conflict hiding equilibrium $E_{LM}(p,k)$, on the other hand, sustains at most k number of partitions.

We show that if $b_h \leq 1/\sqrt{24}$, $E(RE, v)$ can never generate a higher welfare for the receiver compared to the LM equilibrium $E_{LM}(p,k)$. For this, we use (a) the condition on p for the existence of an equilibrium, and (b) the condition on p that ensures that the number of partitions in the $E_{LM}(p,k)$ is less than the number of partitions that RE can sustain. We find out the range of b_h for which these two conditions can never be satisfied simultaneously.

For $E(RE,v)$ to exist, we get get a range of p from the IC for b_l type:

$$p \leq p_2 = \frac{(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)) - (CS_{b_h}^{b_l}(m) + A)}{(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m))} \tag{29}$$

From quation 7 in appendix A, we know that $E_{LM}(p,k)$ can sustain at most $n-1$ partitions only when:

$$p \geq \underline{p} = \frac{\frac{1}{2n(n-1)} - b_l}{b_h - b_l} \tag{30}$$

This implies that if $p_2 < \underline{p}$, conditions 29 and 30 can not hold together, so $E(RE,v)$ can never be better than $E_{LM}(p.k)$ in terms of the receiver's utility.

If $\underline{p} > p_2$, that is,

$$\frac{\frac{1}{2n(n-1)} - b_l}{b_h - b_l} > 1 - \frac{CS^{b_l b_h}(m) + A}{CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)}$$

$$\frac{CS_{b_h}^{b_l}(m) + A}{CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)} > \frac{b_h - \frac{1}{2n(n-1)}}{b_h - b_l}$$

$$\left(\frac{m^2-1}{12m^2} + b_l^2 - \frac{(b_h+b_l)^2}{2})(b_h - b_l\right) > \left(b_h - \frac{1}{2n(n-1)}\right)\left(\frac{n^2-m^2}{12m^2n^2} - \frac{b_l^2(n^2+2)}{3} + \frac{(b_h+b_l)^2}{2}\right)$$

Note that, as b_h increases, i.e., m decreases, p_2 increases (from lemma 2), but $\underline{p}$ decreases, so $\underline{p} - p_2$ decreases. Thus at the minimum possible value of m, namely $m = 2$, $\underline{p} - p_2$ would be minimized. For $m = 2$ the inequality $\underline{p} > p_2$ can be written as:

$$\left(\frac{1}{16} + b_l^2 - \frac{(b_h+b_l)^2}{2}\right)(b_h - b_l) > \left(b_h - \frac{1}{2n(n-1)}\right)\left(\frac{n^2-4}{48n^2} - \frac{b_l^2(n^2+2)}{3} + \frac{(b_h+b_l)^2}{2}\right)$$

Let us define the difference $D \equiv \underline{p} - p_2$ as follows:

$$D = (b_h - b_l)\left(\frac{1}{16} + b_l^2 - \frac{(b_h+b_l)^2}{2}\right) - \left(b_h - \frac{1}{2n(n-1)}\right)\left(\frac{n^2-4}{4bn^2} - \frac{b_l^2(n^2+2)}{3} + \frac{(b_h+b_l)^2}{2}\right)$$

We want to find conditions on b_h such that $D > 0$ i.e., $\underline{p} > p_2$. Since $\underline{p} - p_2$ is minimized at $m = 2$, if $D > 0$ for $b_h \leq \bar{b}$ where $\bar{b} \in (1/12, 1/4)$ then $\underline{p} > p_2$ for all $m > 2$.

$$\left(\frac{1}{16} + b_l^2 - \frac{(b_h + b_l)^2}{2}\right)(b_h - b_l) > \left(b_h - \frac{1}{2n(n-1)}\right)\left(\frac{n^2-4}{48n^2} - \frac{b_l^2(n^2+2)}{3} + \frac{(b_h+b_l)^2}{2}\right)$$

Our goal is to find the range of b_h for which $D > 0$. We show that $\frac{\partial D}{\partial b_l} < 0$, i.e, D is a decreasing function of b_l and at the highest value of b_l given n, $D > 0$ and hence it will be positive for any lower b_l as well.

First, we show that $\frac{\partial^2 D}{\partial b_l^2} > 0$ for all b_l. Next, we find that $\frac{\partial D}{\partial b_l} < 0$ at the highest values of b_l given n. Hence, $\frac{\partial D}{\partial b_l} < 0$ for all values of b_l.

The derivative of D wrt b_l

$$\frac{\partial D}{\partial b_l} = -\frac{1}{16} - \frac{3(b_h^2 + b_l^2)}{2} + \frac{2b_h b_l (n^2+5)}{3} - \frac{b_l(2n^2+1)}{6n(n-1)} + \frac{b_h}{2n(n-1)}$$

and the second derivative would be,

$$\frac{\partial^2 D}{\partial b_l^2} = -3b_l + \frac{2b_h(n^2+5)}{3} - \frac{2n^2+1}{6n(n-1)}.$$

Since $\frac{\partial^2 D}{\partial b_l^2}$ is increasing in n and b_h we check that at the min possible value of n, namely $n = 3$ and b_h, namely $b_h = 1/12$ (given $m = 2$):

$$\frac{\partial^2 D}{\partial b_l^2} = -3b_l + \frac{1}{4} > 0$$

for all $b_l < \frac{1}{12}$. Since $\frac{\partial^2 D}{\partial b_l^2} > 0$ for $n \geq 3$, we will show that at the maximum possible value of b_l: $b_l \to 1/2n(n-1)$, where $\frac{\partial D}{\partial b_l}$ is maximized, $\frac{\partial D}{\partial b_l} < 0$. Now,

$$\lim_{b_l \to 1/2n(n-1)} \frac{\partial D}{\partial b_l} = -\frac{1}{16} - \frac{3b_h^2}{2} - \frac{4n^2+11}{24n^2(n-1)^2} + b_h \frac{2n^2+13}{6n(n-1)} \leq 0$$

for $n \geq 4$. For $n = 3$, however, there exists range of b_h such that whenever $b_h > b_h' > 1/12$, $\lim_{b_l \to 1/2n(n-1)} \frac{\partial D}{\partial b_l} > 0$. Next, we find out that threshold b_h'.

Rewrite $D \geq 0$ as,

$$(b_h - b_l)(\frac{1}{16} + b_l^2 - \frac{(b_h+b_l)^2}{2}) \geq (b_h - \frac{1}{2n(n-1)})(\frac{n^2-4}{4bn^2} - \frac{b_l^2(n^2+3)}{3} + \frac{(b_h+b_l)^2}{2})$$
$$\Rightarrow \frac{n^2+2}{6} + \frac{n^2+5}{3(n-1)^2} \geq \frac{(b_h 2n(n-1)+1)^2}{(n-1)^2}$$
$$\Rightarrow n^4(1-24b_h^2) + 2n^3(24b_h^2-1) + n^2(5-24b_h^2+24b_h) + n(24b_h-4) + 6 \geq 0$$

Consider the case $n \geq 4$ .For $(1-24b_h^2) > 0$ or $b_h < 1/\sqrt{24}$, the leading term of the polynomial is positive. and hence the polynomial is positive for all $n \geq 4$.

Now let us consider the case for $n = 3$. At $b_h = 1/\sqrt{24}$, we find,

$$\frac{\partial D}{\partial b_l} \approx -7 < 0$$

And, we have already shown that $\frac{\partial^2 D}{\partial b_l^2} > 0$, Thus for all $b_h \leq 1/\sqrt{24}$, $\frac{\partial D}{\partial b_l} < 0$. Hence, as argued before, D is mininized as $b_l \to 1/2n(n-1)$. So, for all $b_h \leq 1/\sqrt{24}$ and n=3, the minimum value of D is:

$$D = n^4(1-24b_h^2) + 2n^3(24b_h^2-1) + n^2(5-24b_h^2+24b_h) + n(24b_h-4) + 6 \geq 0$$

Thus, for all $n \geq 3$ we find that for $b_h \leq 1/\sqrt{24}$ the LM equilibrium generates higher welfare for the receiver compared to the RE equilibrium.

If $b_h \in (\frac{1}{\sqrt{24}}, \frac{1}{4})$, there exists an $E(RE, v)$ that generates a higher welfare for the receiver compared to the LM equilibrium.

□

Proof of proposition 4

Proof. Consider the following strategies. Both types of senders reveal their type. For the receiver's strategy, we abuse notation and write the equilibrium she will play:

receiver strategy:

If $\mu_b = b_l$, with probability (v_l), play $E_{CS}(b_l, j)$ and with probability $(1-v_l)$, play $E_{CS}(b_l, k)$

If $\mu_b = b_h$, with probability (v_h), play $E_{CS}(b_h, x)$ and with probability $(1-v_h)$, play $E_{CS}(b_h, y)$

where $j, k, x, y \in \mathbb{N}$ and $v_l, v_h \in [0,1]$. WLOG let $j \geq k, x \geq y$

Suppose such an equilibrium exists for some choice of parameters. The IC conditions for truth-telling is give by,

$$IC_{b_l} : \; v_l CS_{b_l}^{b_l}(j) + (1-v_l)CS_{b_l}^{b_l}(k) \geq v_h CS_{b_h}^{b_l}(x) + (1-v_h)CS_{b_h}^{b_l}(y) \tag{31}$$

$$IC_{b_h} : \; v_h CS_{b_h}^{b_h}(x) + (1-v_h)CS_{b_h}^{b_h}(y) \geq v_l CS_{b_l}^{b_h}(j) + (1-v_l)CS_{b_l}^{b_h}(k) \tag{32}$$

Adding, we get,

$$\begin{aligned} v_l(CS_{b_l}^{b_l}(j) - CS_{b_l}^{b_h}(j)) + (1-v_l)(CS_{b_l}^{b_l}(k) - CS_{b_l}^{b_h}(k)) \geq \\ v_h(CS_{b_h}^{b_l}(x) - CS_{b_h}^{b_h}(x)) + (1-v_h)(CS_{b_h}^{b_l}(y) - CS_{b_h}^{b_h}(y)) \end{aligned} \tag{33}$$

Using equations 17, 18 and the corresponding expressions for the low bias sender, we can calculate:

$$CS_{b_l}^{b_l}(n) - CS_{b_l}^{b_h}(n) = (b_h - b_l)\left(b_h \frac{1}{n} + b_l(2 - \frac{1}{n})\right) \tag{34}$$

$$CS_{b_h}^{b_l}(m) - CS_{b_h}^{b_h}(m) = (b_h - b_l)\left(b_h(2 - \frac{1}{m}) + b_l \frac{1}{m}\right) \tag{35}$$

Plugging in the values from above in 33 we get,

$$(b_h - b_l)\left(\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} - 2\right) \geq 0 \tag{36}$$

Since $b_h \geq b_l$ this would be true if and only if

$$\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} \geq 2 \tag{37}$$

which would require $j,k,x,y \leq 1$. This would imply the equilibrium play would be babbling under all possible revelation in the first stage. However, this is not true since we were looking for an informative bias revealing equilibrium. Therefore our assumption is wrong and there does not exist any informative equilibrium in the one sender case even if we permit the receiver to mix with commitment in hiring. □

Proof of proposition 5

Proof. IC for b_h:

$$p\left(v-\frac{1}{2}\right)\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right) > \frac{1}{2}\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)+\left(\frac{1}{2}-v\right)\left(CS_{b_h}^{b_h}(m)+A+c\right) \quad (38)$$

For $v < \frac{1}{2}$, we can rewrite the IC as follows:

$$p < \frac{\frac{1}{2}\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)+\left(\frac{1}{2}-v\right)\left(CS_{b_h}^{b_h}(m)+A+c\right)}{\left(v-\frac{1}{2}\right)\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)} \quad (39)$$

Note that since $CS_{b_l}^{b_h}(n) \geq CS_{b_h}^{b_h}(m)$ as shown in lemma 2 the denominator is negative. Also, for any $c \geq 0$, $CS_{b_h}^{b_h}(m)+A+c > 0$ thus the numerator is positive. This implies equation 39 holds only if $p < 0$. Thus $v < \frac{1}{2}$ cannot be a possible equilibrium.

On the other hand if $v \geq \frac{1}{2}$, we can rewrite the IC as,

$$p > \frac{\frac{1}{2}\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)+\left(\frac{1}{2}-v\right)\left(CS_{b_h}^{b_h}(m)+A+c\right)}{\left(v-\frac{1}{2}\right)\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)} = p_1 \quad (40)$$

We need that the RHS is less than 1 to get a feasible region for p.We know that the denominator is positive, since $CS_{b_l}^{b_h}(n) \geq CS_{b_h}^{b_h}(m)$ as shown in lemma 2. In the numerator, since $CS_{b_l}^{b_h}(n) \geq CS_{b_h}^{b_h}(m)$, and $CS_{b_h}^{b_h}(m) > -(A+c)$, the first expression is positive while the second expression is negative. Then, for every $v \in (1/2,1]$, $\exists c_1(v) = \frac{(v+1)\left(CS_{b_l}^{b_h}(n)-CS_{b_h}^{b_h}(m)\right)}{\left(v-\frac{1}{2}\right)} - CS_{b_h}^{b_h}(m)-A$. Thus, $\forall c > c_1(v), RHS < 1$.

Considering $v > \frac{1}{2}$, the IC for b_l is given as below

$$p < \frac{\frac{1}{2}\left(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)\right) + \left(\frac{1}{2} - v\right)\left(CS_{b_h}^{b_l}(m) + A\right)}{\left(v - \frac{1}{2}\right)\left(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)\right)} = p_2 \tag{41}$$

We need the RHS to be greater than zero to get a feasible region for p. Since $CS_{b_l}^{b_l}(n) > CS_{b_h}^{b_l}(m))$ (refer lemma 2), the denominator is positive. Furthermore, the first expression of the numerator is positive while the second is negative. At $v = \frac{1}{2}, RHS > 0$, and RHS is continuous and decreasing in v, so there must exist a $\overline{v} = \frac{1}{2}\frac{CS_{b_l}^{b_l}(n)+A}{CS_{b_h}^{b_l}(m)+A}$, such that RHS=0. Since $CS_{b_l}^{b_l}(n) > CS_{b_h}^{b_l}(m)$, this $\overline{v} > \frac{1}{2}$. Thus, for all $\frac{1}{2} < v < \overline{v}, RHS > 0$.

To satisfy both the ICs simultaneously, we need:

$$\frac{\frac{1}{2}(CS_{b_l}^{b_h}(n) - CS_{b_h}^{b_h}(m)) + (\frac{1}{2} - v)(CS_{b_h}^{b_h}(m) + A + c)}{(v - \frac{1}{2})(CS_{b_l}^{b_h}(n) - CS_{b_h}^{b_h}(m))} < \frac{\frac{1}{2}(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)) + (\frac{1}{2} - v)(CS_{b_h}^{b_l}(m) + A)}{(v - \frac{1}{2})(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m))}$$

For any v' such that $\frac{1}{2} < v' < \overline{v}$ we can always choose c high enough to make the above inequality hold. Suppose it holds when $c > c_2(v')$. Now for this v', we can find $c_1(v')$ which makes the IC for b_h feasible.

Then, for every $\frac{1}{2} < v' < \overline{v}$, we can find a $\overline{c}(v') = max\{c_1(v'), c_2(v')\}$ such that given any v' for all $c > \overline{c}(v')$, our bias revealing strategy profile described in 2 is an equilibrium.

□

Proof of proposition 6

Proof. Let $N_{b_l} = n$, $N_{b_h} = m$ and the maximum number of partitions possible in $E_{LM}(p,k)$ is $k \geq 2$.

Then, we know from proposition 5, that there exists a $\bar{v}$ such that given any $v \in [\frac{1}{2}, \bar{v}]$, there exists a $\bar{c}(v)$ such that if $c > \bar{c}(v)$ the bias revealing strategies described in 2 constitute an equilibrium.

Following equation 15 the difference between the payoffs of the receiver from $E(RE, v)$ and

$E_{LM}(p,k)$ as derived in appendix A can be written as a quadratic expression:

$$\Delta(p) = Ap^2 + Bp + C$$

$$where$$

$$A = (CS_{b_h}^{b_h}(m) - CS_{b_l}^{b_l}(n))(1-2v) + (k-1)(b_h - b_l)^2(\frac{k+1}{3} - \frac{1}{k})$$

$$B = (CS_{b_h}^{b_h}(m) - CS_{b_l}^{b_l}(n))v + (k-1)(b_h - b_l)(\frac{(k+1)b_l}{3} + \frac{b_h - b_l}{2k})$$

$$C = CS_{b_l}^{b_l}(n) - CS_{b_l}^{b_l}(k)$$

We need to show that $\Delta(p) > 0$ for a range of p. The roadmap of this proof is as follows. First, we note that for the existence of an equilibrium, we need $p \in [p_1, p_2]$, i.e., the ICs for both types of senders are satisfied, as given in 40 and 41. For a sufficiently low b_l, we can choose c high enough such that p_1 goes to zero, so for a low p we know that equilibrium exists.

Second, for very low p, whenever $C > 0$, we get $\Delta > 0$, so next we find conditions for which $C > 0$. This gives us another lower bound on p: $p > z$.

Third, for $b_l \to 0$, we can show that $z \to 0$, and there exists a threshold of v, $v^* > \frac{1}{2}$ such that at $p = z$, the $E(RE, v)$ exists and $\Delta > 0$. Hence, for all $p \in [z, p_2]$, $E(RE, v)$ is better than $E_{LM}(p,k)$ equilibria.

For the first step, we need to ensure the existence of the bias-revealing equilibrium, that requires $p \in I = [p_1, p_2]$ (given by 40 and 41 in Proposition 4). Now, let us pick a $v \in [\frac{1}{2}, \bar{v}]$. From Proposition 4, we know that there exists a $\bar{c}(v)$ such that $\forall c > \bar{c}(v)$, 40 holds, so let us pick a $c = c' > \bar{c}$ which makes $p_1 \to 0$.

For the second step, since the function is quadratic in p, if $C > 0$, then for a very low p, the difference in payoffs: $U_R(RE, v) - U_R(LM, p, k)$ is positive. $C > 0$ requires that the maximum number of partitions in the conflict hiding equilibrium is strictly less than the maximum number of partitions possible when the bias of the sender is known to be b_l i.e. $n > k$. By lemma 1 this requires

$$p > z = \frac{\frac{1}{2n(n-1)} - b_l}{b_h - b_l} \tag{42}$$

So, for the existence of a revealing equilibrium where $n > k$, we need

$$p > max(z, p_1) \tag{43}$$

where p_1 is given in 39.

Thus, $max(z, p_1) = z$, so we only need to show that at $p = z, \Delta > 0$. For this to be an equilibrium, we also need IC for b_l to hold. Using 41, we need:

$$z < p_{h_1} = \frac{\frac{1}{2}(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m)) + \frac{1}{2}(CS_{b_h}^{b_l}(m) + A) - v(CS_{b_h}^{b_l}(m) + A)}{(v - \frac{1}{2})(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_l}(m))} \tag{44}$$

Let $b_l \to 0$. Then, $n \to \infty$, hence $z \to 0$ from above, hence satisfies 44.

At $p = z$,

$$Ap^2 + 2Bp + C > 0 \;\Rightarrow v < \overline{v} \tag{45}$$

$$where\ \overline{v} = \frac{z^2 CS_{b_h}^{b_h}(m) + (1-z)^2 CS_{b_l}^{b_l}(n) - CS_{b_z}^{b_z}(k) + \frac{(k-1)(b_h - b_l)^2 z(1-z)}{k}}{2z(1-z)(CS_{b_l}^{b_l}(n) - CS_{b_h}^{b_h}(m))}$$

$$b_z = zb_h + (1-z)b_l \tag{46}$$

$$and\ CS_{b_z}^{b_z}(k) = -\frac{1}{12k^2} - \frac{b_z^2(k^2-1)}{3} - b_z^2$$

As $b_l \to 0$, 45 holds because $\overline{v}$ goes to ∞. For 44 to hold, we get the condition:

$$v < \frac{1}{2}\left(\frac{A}{CS_{b_h}^{b_l}(m) + A}\right) \tag{47}$$

Thus, we can find a $v^*(> \frac{1}{2})$ such that 44 holds if $\frac{1}{2}\left(\frac{A}{CS_{b_h}^{b_l}(m)+A}\right) > \frac{1}{2}$. This is true because the highest possible payoff for the receiver at any state is zero and therefore $CS_{b_h}^{b_l}(m) < 0$.

Thus, there exists a cutoff $\bar{b}$ such that if $b_l < \bar{b}$ then there exists a set G of the form $[z, p_{h_1}]$, a $v^* > \frac{1}{2}$ and a c^* such that the bias revealing strategies in 5 constitute an equilibrium and they give the receiver a higher payoff than any payoff possible in a conflict hiding equilibrium without the bias revelation stage. Hence, proved.

□

C Partially Conflict Revelation Equilibrium

As outlined in Li and Madarász (2008) another type of equilibrium can exist without the bias revelation stage. In such an equilibrium, the low-bias sender can reveal his type but only up to the state $s \leq d \equiv b_h - b_l$. The equilibrium structure takes the following form: In the space $[0,s]$, we have $E_{CS}(b_l, j)$, where the b_l type sender's type is revealed. In $[s,1]$, we have a conflict hiding equilibrium with k partitions.

Let $\alpha^l_1, \dots \alpha^l_j$ denote the cutoffs in the $[0,s]$ interval. Let the corresponding actions chosen by the decision-maker be $y^l_1, \dots y^l_j$ respectively. Since the bias is revealed in this case, the receiver will choose actions y^l_i a la Crawford and Sobel (1982). Thus we will get,

$$\alpha_i = \alpha_1 i + 2i(i-1)b_k$$

Since $\alpha^l_j = s$ we get,

$$\alpha_1 = \frac{s}{j} - 2(j-1)b_l; \quad \Rightarrow \quad \alpha_i = \frac{si}{j} - 2i(j-i)b_l$$

The utility of the receiver from this conflict revealing messages are given by,

$$U_R(CR, j, s) = -\frac{1}{12}\sum_{i=1}^{j}(\alpha_i - \alpha_{i-1})^3 = -\frac{s^3}{12j^2} - \frac{sb_l^2(j^2-1)}{3}$$

When the type of the player is not revealed, suppose b_h chooses $a^h_1, \dots a^h_k$ as the cutoffs over $[0,1]$ and b_l chooses $a^l_1, \dots a^l_k$ as the cutoff for the same messages over $[s,1]$. Let $y_1, \dots y_k$ denote the corresponding action chosen by the receiver for these k messages. Then,

$$y_1 = p\frac{a^h_1}{2} + (1-p)\frac{a^l_1 + s}{2}$$
$$y_i = p\frac{a^h_i + a^h_{i-1}}{2} + (1-p)\frac{a^l_i + a^l_{i-1}}{2}$$
$$y_k = p\frac{1 + a^h_{k-1}}{2} + (1-p)\frac{+a^l_{k-1}}{2}$$

Taking differences we get, for all $i = 2, \ldots k$

$$y_i - y_{i-1} = 2y_1 + 2b + (i-2)4b - 2(1-p)s; \; where \; b = pb_h + (1-p)b_l$$

Combining this with,

$$1 - y_k = y_1 + 2b(k-1) - (1-p)s$$

we get,

$$\begin{aligned} y_1 &= \frac{1}{2k} - b(k-1) + \frac{2k-1}{2k}(1-p)s \\ y_i &= \frac{2i-1}{2k} + b(2i^2 - (k+1)(2i-1)) + \frac{2k-2i+1}{2k}(1-p)s \\ y_k &= \frac{2k-1}{2k} - b(k-1) + \frac{1}{2k}(1-p)s \end{aligned}$$

The receiver's payoff if being matched with b_h sender would be

$$\begin{aligned} U_R^h(CH,s,k) &= \int_0^{a_1^h} -(y_1-\theta)^2 d\theta + \int_{a_1^h}^{a_2^h} -(y_2-\theta)^2 d\theta + \cdots + \int_{a_{k-1}^h}^{1} -(y_k-\theta)^2 d\theta \\ &= \frac{1}{3}\left[\left[(y_1-\theta)^3\right]_0^{a_1^h} + \left[(y_2-\theta)^3\right]_{a_1^h}^{a_2^h} + .. + \left[(y_k-\theta)^3\right]_{a_{k-1}^h}^{1}\right] \\ &= \frac{1}{3}\left[-(1-y_k)^3 - y_1^3 + \sum_{i=1}^{k-1}\left(\left(y_i - a_i^h\right)^3 - \left(y_{i+1} - a_i^h\right)^3\right)\right] \end{aligned}$$

Plugging in the values of y_i and a_i^h we get,

$$\begin{aligned} (y_i - a_i^h)^3 - (y_{i+1} - a_i^h)^3 &= (-\frac{1}{2k} + b(k-2i) + \frac{(1-p)s}{2k} + b_h)^3 - (\frac{1}{2k} - b(k-2i) - \frac{(1-p)s}{2k} + b_h)^3 \\ &= -2(\frac{1}{2k} - b(k-2j) - \frac{(1-p)s}{2k})^3 - 6b_h^2(\frac{1}{2k} - b(k-2j) - \frac{(1-p)s}{2k}) \end{aligned}$$

Hence, we get,

$$\begin{aligned} U_R^h(CH,s,k) = -\frac{1}{3}[&\frac{(1-(1-p)s)^3(k-1)}{4k^3} + (1-(1-p)s)(k-1)(b^2(k-2) + \frac{3b_h^2)}{k} \\ &+ (1-y_k)^3 + y_1^3] \end{aligned}$$

Similarly, the receiver's payoff if being matched with b_l sender would be

$$\begin{aligned}
U_R^l(CH,s,k) &= \int_s^{a_1^l} -(y_1-\theta)^2 d\theta + \int_{a_1^l}^{a_2^l} -(y_2-\theta)^2 d\theta + \cdots + \int_{a_{k-1}^l}^{1} -(y_k-\theta)^2 d\theta \\
&= \frac{1}{3}\left[\left[(y_1-\theta)^3\right]_s^{a_1^l} + \left[(y_2-\theta)^3\right]_{a_1^l}^{a_2^l} + .. + \left[(y_k-\theta)^3\right]_{a_{k-1}^l}^{1}\right] \\
&= \frac{1}{3}\left[-(1-y_k)^3 - (y_1-s)^3 + \sum_{i=1}^{k-1}\left(\left(y_i-a_i^l\right)^3 - \left(y_{i+1}-a_i^l\right)^3\right)\right]
\end{aligned}$$

and similarly, we can show that,

$$\begin{aligned}
U_R^l(CH,s,k) = &-\frac{1}{3}[\frac{(1-(1-p)s)^3(k-1)}{4k^3} + (1-(1-p)s)(k-1)(b^2(k-2)+\frac{3b_l^2}{k}) \\
&+(1-y_k)^3 + (y_1-s)^3]
\end{aligned}$$

Thus the expected payoff of the receiver from the conflict hiding region of the equilibrium would be,

$$\begin{aligned}
U_R(CH,s,k) = &-\frac{1}{3}(\frac{(1-(1-p)s)^3(k-1)}{4k^3} + b^2(1-(1-p)s)(k-1)(k-2)) \\
&-(1-(1-p)s)\frac{k-1}{k}(pb_h^2 + (1-p)b_l^2) - \frac{1}{3}((1-y_k)^3 + py_1^3 + (1-p)(y_1-s)^3)
\end{aligned}$$

where,

$$\begin{aligned}
(1-y_k)^3 + py_1^3 + (1-p)(y_1-s)^3 = &(\frac{1-(1-p)s}{2k} - b(k-1))^3 \\
&+p(\frac{1-(1-p)s}{2k} - b(k-1) + (1-p)s)^3 \\
&+(1-p)(\frac{1-(1-p)s}{2k} - b(k-1) - ps)^3
\end{aligned}$$

Simplifying we get,

$$\begin{aligned}
(1-y_k)^3 + py_1^3 + (1-p)(y_1-s)^3 = &2(\frac{1-(1-p)s}{2k})^3 + 6\frac{1-(1-p)s}{2k}b^2(k-1)^2 \\
&+s^3p(1-p)(1-2p) + 3s^2p(1-p)(\frac{1-(1-p)s}{2k} - b(k-1))
\end{aligned}$$

Plugging this value in the receiver's payoff we get,

$$
\begin{aligned}
U_R(CH,s,k) = &-\frac{1}{3}(\frac{(1-(1-p)s)^3(k-1)}{4k^3}+b^2(1-(1-p)s)(k-1)(k-2)) \\
&-(1-(1-p)s)\frac{k-1}{k}(pb_h^2+(1-p)b_l^2) \\
&-\frac{1}{3}(2(\frac{1-(1-p)s}{2k})^3+6\frac{1-(1-p)s}{2k}b^2(k-1)^2 \\
&+s^3p(1-p)(1-2p)+3s^2p(1-p)(\frac{1-(1-p)s}{2k}-b(k-1)))
\end{aligned}
$$

Simplifying we get,

$$
\begin{aligned}
U_R(CH,s,k) = &-\frac{1}{12k^2}(1-(1-p)s)^3-\frac{b^2}{3}(k^2-1)(1-(1-p)s) \\
&-d^2p(1-p)\frac{k-1}{k}(1-(1-p)s) \\
&-\frac{1}{3}s^2p(1-p)(s(1-2p)+2(\frac{1-(1-p)s}{2k}-b(k-1)))
\end{aligned}
$$

So the total payoff of the receiver would be given by,

$$
\begin{aligned}
U_R(LM,p,s,j,k) = &-\frac{s^3}{12j^2}-\frac{sb_l^2(j^2-1)}{3} \\
&-\frac{1}{12k^2}(1-(1-p)s)^3-\frac{b^2}{3}(k^2-1)(1-(1-p)s) \\
&-d^2p(1-p)\frac{k-1}{k}(1-(1-p)s) \\
&-\frac{1}{3}s^2p(1-p)(s(1-2p)+2(\frac{1-(1-p)s}{2k}-b(k-1)))
\end{aligned}
$$

Proof under reconstruction