

Signalling, Reputation and Spinoffs

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Internet Appendix

This internet appendix contains some proofs which were not included in the main body of the paper. Additionally, it contains other results of interest which could not be included in the paper in order to keep the paper of readable length. It is organized as follows. In section 1, I discuss proofs which were omitted from the main body of the paper. In particular, I discuss the case where the worker's type is common knowledge in the baseline model. Section 2 contains results which were not presented in the moral hazard section of the paper - I describe conditions under which the separating equilibrium may not be highest effort equilibrium. In section 3, I discuss conditions under which the separating equilibrium outcome (as described by proposition 1 in the main body of the paper) will be the unique equilibrium outcome of the game.

1 Proof from main body of paper

In the main body of the paper we did not derive the equilibrium when the worker type was common knowledge in the baseline model. We do so here and show that the worker never forms a spinoff when his type is known. I will solve the one period game beginning in period 2 first and then proceed by backwards induction. Since firm formation is too costly in period two¹, it is trivial to show that the following is true in any equilibrium. If the worker formed his own firm in period one, then he will continue to remain with his own firm in period 2, and if the worker accepted the principal's contract in period one then the principal would offer a zero wage contract in period 2 which will be accepted by the worker. Thus, the payoffs in period 2 in any equilibrium can be summarized as follows (first coordinate is principal's payoffs and the second is worker's payoff):

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¹Both the principal and the worker do not want to invest in firm formation if the reward is just a one period payoff. The assumption of $R_w > R_p > V$ reflects this.

Table 1: Payoffs in Period 2

Table 2: Payoff if contract accepted in Period 1

$(Principal, G)$	$(Principal, B)$
$(V, 0)$	$(\lambda_b V, 0)$

Table 3: Payoff if Spinoff in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, V)$	$(0, \lambda_b V)$

Table 4: Payoff if N in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, 0)$	$(0, 0)$

1.0.1 In Period 1

Claim 1. *In the unique subgame perfect equilibrium of the game, if the worker type is B , the principal offers the worker a contract with zero wages $(\{0, 0\})$ in period 1 and the worker accepts. If the worker type is G then the principal offers the contract $\{2V - R_w, 0\}$ in period 1 and it is accepted by the worker. In period 2, (along the equilibrium path) the principal offers a zero wage contract which is accepted by the worker.*

Proof. By assumption $(2\lambda_b V < R_w)$ it is not individually rational (IR) for B worker to invest in forming own firm in period 1. Therefore, the principal will offer a zero wage contract which will be accepted. Note that it is IR for the principal to offer this contract since $2\lambda_b V > R_p$. It will not be incentive compatible to offer more. When the worker type is G , the worker gets the payoff $(V - R_w + V)$ by investing R_w and starting his own firm. The principal must offer him this amount to dissuade him from leaving. Offering more will not be optimal. Moreover, it is known that the principal cannot commit to offering more than zero wages in period 2 if the worker does not form a firm in period 1. Therefore, the principal offers him a contract which pays him $(2V - R_w)$ upon success². Offering this contract is also IR for the principal as the principal's payoff is $V - (2V - R_w) - R_p + V = R_w - R_p (> 0)$. \square

Corollary 1. *The worker never forms his own firm if worker type is known.*

Thus, equilibrium payoffs to the principal and worker in the game when types are known are given by the following matrix where (the first coordinate refers to the principal's payoff)

Table 5: Payoffs if Types are Known

$(Principal, G)$	$(Principal, B)$
$(R_w - R_p, 2V - R_w)$	$(2\lambda_b V - R_p, 0)$

²Since G type always succeeds, there is no point in offering him any payoff for failure.

2 Additional Results from Moral Hazard Section

The claims (claim 2 and claim 3) below serve to show that the spinoff equilibrium may not always produce the most effort.

Claim 2. *There exists \bar{R} such that if $R_w > \bar{R}$ and the following holds*

$$\frac{\frac{1+\beta}{1-\beta} + R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)} < R_p$$

then there exists p' such that if $p_g < p'$, there exists a separating equilibrium where the G worker's strategy (along the equilibrium path) is to leave and form his own firm in period 1 and the B worker's strategy (along the equilibrium path) is to accept a contract in period 1. B type worker exerts $e = 1$ in both periods and G worker exerts $e = 1$ in period 1 and $e = 0$ in period 2. The principal offers the contract $\{(1/(1-\beta)), 0\}$ to the worker in period 1 and the same in period two if the worker accepts in period 1.

Proof. Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(acc, p_g^2) = \left\{ \frac{1}{1-\beta}, 0 \right\}; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(acc, p_g^2) = \{0, 0\}; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

For Worker :

If worker had formed firm in period 1 only one case is relevant since $V < R_p$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, e = 0$$

If worker had not formed firm in period 1 :

$$s_{w,2}(acc/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(acc/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(acc/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi) = \left\{ \left\{ \frac{1}{1-\beta}, 0 \right\} \right\}$$

For Worker :

WLOG let contract 1 be better than contract 2 for G worker

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 1; -1 + s_1^1 + \frac{1}{1-\beta} = \max\{-1 + s_1^1 + \frac{1}{1-\beta}, \beta s_1^1 + \frac{1}{1-\beta}\} \text{ and}$$

$$-1 + s_1^1 + \frac{1}{1-\beta} \geq -1 + V - R_w + \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 0; -1 + s_1^1 + \frac{1}{1-\beta} \neq \max\{-1 + s_1^1 + \frac{1}{1-\beta}, \beta s_1^1 + \frac{1}{1-\beta}\} \text{ and}$$

$$\beta s_1^1 + \frac{1}{1-\beta} \geq -1 + V - R_w + \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = L, e = 1; \text{ else.}$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = L, e = 1$$

The belief about the worker's type are formed using Bayesian updation based on equilibrium strategies. On off equilibrium paths, I will assume beliefs consistent with type-independent trembles in the worker's decision³. Let us prove by backward induction that the above strategies and beliefs constitute a perfect bayesian equilibrium under the conditions imposed in the theorem.

In period 2, consider the incentives of the worker. If the history is such that the worker has formed a firm of his own in period 1, then the worker is not offered any contracts in period 2 since the principle can never find this to be individually rational as $V < R_p$. In this case, it is clear that the best action for the worker is to play S and then exert zero effort. Zero effort is exerted since all payments are received by the worker before his effort choice and the game ends in period 2. If the history is one in which the worker had played N in period 1, then it is not individually rational for either the worker or the principal to form a firm in period 1, therefore the only on the equilibrium path response is to play N again. In a history in which the worker had accepted a contract in period 1, it is not individually rational for the worker to think about playing L , therefore the best action choice is to choose the most profitable (contract, effort) tuple.

Now let us consider the incentives of the principal in period 2. Suppose the history is one in which the worker did not accept the principal's contract in period 1. It is not IR for the principal to form the firm in period 2. Therefore the principal's optimal action is to offer no contract in period 2. If the worker had accepted a contract with the principal in period 1, then the principal has two choices - he can either offer a

³If the worker wants to choose an action a , then the worker chooses action a with probability $(1 - \varepsilon)$ and any other action with positive probability.

contract $\{(1/(1-\beta)), 0\}$ which is the minimum amount required for a worker of either type to put in effort 1^4 or he can offer a contract which pays nothing and get zero effort from the worker. Note that any contract which offers something in the middle is not optimal. We need the following conditions to make it IR and IC for the principal to offer the contract $\{(1/(1-\beta)), 0\}$ and extract maximum effort:

$$\begin{aligned}
& \text{Payoff from } \left\{ \frac{1}{1-\beta}, 0 \right\} > \text{Payoff from } \{0, 0\} \\
& \Leftrightarrow [p_g^2 + (1-p_g^2)(\beta\lambda_b + (1-\beta))](V - \frac{1}{1-\beta}) > [p_g^2\beta + (1-p_g^2)\beta\lambda_b]V \\
& \Leftrightarrow p_g^2 \leq \frac{V(1-\beta)^2 - \beta\lambda_b - (1-\beta)}{\beta(1-\lambda_b)} \tag{1}
\end{aligned}$$

It is clear that if R_w is high enough and $V > (((1+\beta)/(1-\beta)) + R_w)/(1+\beta)$, then $V(1-\beta)^2 - \beta\lambda_b - (1-\beta) > 0$. In any separating equilibrium where the G worker forms his own firm in period 1, $p_g^2 \approx 0$. Thus, 1 is satisfied.

Therefore, under a separating equilibrium, the payoffs in period 2 are as follows. The principal expects to get a payoff of $(\beta\lambda_b + 1 - \beta)(V - (1/(1-\beta)))$ in period 2 if the worker accepts the contract in period 1. If the worker does not accept contract in period one, the principal expects zero payoff in period 2. A G type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1 + (1/(1-\beta))$ if he had accepted a contract in period 1. A B type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1 + (\beta\lambda_b + (1-\beta))\frac{1}{1-\beta}$ if he had accepted a contract in period 1.

Let us now consider the incentives for the worker in period 1. Given the payoffs in period 2, G type worker cannot do better. In any separating equilibrium the G worker must put in maximum effort in period 1 after playing L . We need the following condition to ensure that this is incentive compatible: $V > \frac{1}{\beta(1-\beta)(1-\lambda_b)}$. Given that the G worker will put in full effort after separation, it is IR for the G type worker to choose to play L if $V > \frac{1+R_w}{1+\beta}$. It is clear that if R_w is high enough and $V > \frac{1+\beta+R_w}{1+\beta}$, then these conditions are satisfied. It is not IR for the B type worker to leave if the following condition holds: $V < \frac{1+R_w}{1+\beta(1-\beta)(1-\lambda_b)^2}$. This holds according to the conditions in this claim, therefore, his strategy to choose the best (contract, effort) combination is optimal. So till now we have that if R_w is high enough and $\frac{1+R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta)(1-\lambda_b)^2}$, then the following hold: 1. IR for G to play L , 2. Not IR for B to play L , 3. If G plays L , then first period effort is 1, 4. B will accept $\{\frac{1}{1-\beta}, 0\}$ and put in effort 1 in both periods and 5 G worker will play $e = 0$ in period 2 after L in period 1.

Consider incentives of G worker in period 1 after the principal offers a contract $\{s, 0\}$.

⁴This is regardless of worker type i.e. even if the worker type was known, for any type of worker, this contract would be an optimal contract to induce maximum effort.

Payoff from accept and $e = 1 = -1 + s + (-1 + \frac{1}{1-\beta})$

Payoff from accept and $e = 0 = \beta s + (-1 + \frac{1}{1-\beta})$

Payoff from $L = -1 + V - R_w + \beta V$

Therefore, G worker will accept and play $e = 1$ if

$$s \geq \frac{1}{1-\beta} \ \& \ s \geq V - R_w + (\beta V - \frac{\beta}{1-\beta})$$

G worker will accept and play $e = 0$ if

$$s < \frac{1}{1-\beta} \ \& \ s \geq \frac{1}{\beta} [-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})]$$

G worker will play L if

$$s < \frac{1}{\beta} [-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})] \ \& \ s < V - R_w + (\beta V - \frac{\beta}{1-\beta})$$

Now:

$$\begin{aligned} V &> \frac{\frac{1+\beta}{1-\beta} + R_w}{1+\beta} \\ \Leftrightarrow V - R_w + (\beta V - \frac{\beta}{1-\beta}) &> \frac{1}{1-\beta} \end{aligned} \quad (2)$$

and we can show that

$$V - R_w + (\beta V - \frac{\beta}{1-\beta}) < \frac{1}{\beta} [-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})] \quad (3)$$

So by 2 and 3, if the principal offers $s = \frac{1}{1-\beta}$ or below, G worker will play L . If R_w is high enough then we have $\frac{\frac{1+\beta}{1-\beta} + R_w}{1+\beta} < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$. So we can pick a V such that $\frac{\frac{1+\beta}{1-\beta} + R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$.

The principal now has three choices which may be optimal. One, offer a zero wage contract which the B type worker will accept and put zero effort. Two, offer the contract $\{\frac{1}{1-\beta}, 0\}$ which the worker will accept if he is B type and put in full effort. Three, offer the contract $\{V - R_w + (\beta V - \frac{\beta}{1-\beta}), 0\}$ which both type workers will accept and put in full effort⁵. The principal offers the contract $\{\frac{1}{1-\beta}, 0\}$. This is less than the minimum needed to attract the G type worker. Clearly, the principal will be willing to offer no more than this if p_g is low enough i.e. if the principal believes that the customers are convinced that the worker must be B type if he accepts a contract then it would not be optimal for the principal a high paying $(V - R_w + (\beta V - \frac{\beta}{1-\beta}))$ is

⁵3 ensures that that offering a contract which would make the G worker accept and put in zero effort is not optimal for the principal.

strictly bigger than $\frac{1}{1-\beta}$) contract to attract both types of workers⁶. Thus, we can find the required p' . The principal does not offer a lower pay contract because V is high enough for him to want to induce high effort. Given this contract, the G worker's optimal choice is to form his own firm in period 1. \square

Claim 3. *Suppose the conditions required for the previous claim hold. Then, there exists a p'' such that if $p_g \in [0, p'']$, there exists a pooling equilibrium in which a worker of any type chooses to accept a contract in period 1 and 2 and puts in effort=1 in both periods.*

Proof. Suppose the belief about the worker's type on off equilibrium paths are the beliefs consistent with type-independent trembles in the worker's decision. Then, if p_g is low and a worker of any type is expected to accept the contract, and a worker plays L , it would be viewed by the market and the worker will have a reputation of p_g . If p_g is low, it would not be individually rational to play L (this follows from $V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$). Therefore, the strategy of both type workers must be to accept the best contract if the belief is that they will.

Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(nf, p_g^2) = \left\{ \frac{1}{1-\beta}, 0 \right\}; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(nf, p_g^2) = \{0, 0\}; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

For Worker :

If worker had formed firm in period 1 only one case is relevant

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, e = 0$$

If worker had not formed firm in period 1 :

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

⁶I omit the formal proof of this part since the idea is the same as that in propositions in the main body of the paper.

Consider the following strategies in period 1:

For Principal :

$$s_p(\phi) = \left\{ \left\{ \frac{1}{1-\beta}, 0 \right\} \right\}$$

For Worker :

$$s_w(\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}, G/B) = \text{Choose (contract, effort) maximizing payoff}$$

$$s_w(\phi, G/B) = N$$

The above strategies would give us the pooling on contract equilibrium. The proof is similar to those we have given before so we refrain from reproducing it here. \square

Corollary 2. *Given worker type, the above pooling equilibrium is weakly better than the separating equilibrium described in claim 2 since it gives a higher probability of success of the project in each period.*

Proof. For any type of worker, the pooling equilibrium guarantees full effort in both periods. Whereas, the separating equilibrium does not give us full effort from the G type worker in period 2. \square

3 Uniqueness of Equilibrium

Is it possible to get the equilibrium outcome in the separating equilibrium⁷ as the unique equilibrium outcome? I show that if the following two assumptions hold along with the conditions required in proposition 1 (in the main body of the paper), then the separating equilibrium outcome is the unique outcome of the game.

Assume the following:

- Assumption A - The worker can make mistakes. Formally, in every period, after the worker makes his action decision, nature moves and assigns a probability to every action in the worker's choice set. If the worker's strategy is to choose the action b , then nature chooses action b with probability $1 - \epsilon$ and all other actions with equal probability (equal probability weights adding up to ϵ).
- Assumption B - Given an equilibrium, if a mistake (non-equilibrium action) gives the worker strictly negative payoffs for all possible beliefs that the market may have about the worker who makes that mistake, then the probability of such a mistake is much lower than the probability of a mistake where the worker may get positive payoffs for some beliefs. In particular, I assume the following mathematical specification⁸ - if a mistake gives the worker strictly negative payoffs for all possible beliefs that the

⁷Where the good type worker forms a spinoff in period 1 and then remains at the spinoff in period 2 while the bad type worker's strategy is to accept a contract and stay with the principal in period 2.

⁸Any other specification where one mistake is an order more likely than the other will also work

market may have about the worker who makes that mistake, then the probability of such a mistake is ε^2 (independent of type). Else, the probability of the mistake is ε (independent of type)

Our equilibrium concept now is extensive form Trembling Hand Perfect Equilibrium. There are several justifications possible for allowing for mistakes. These could be genuine mistakes, experimentation, unobserved shocks etc. Furthermore, allowing for mistakes tests our equilibrium for robustness.

If Assumption A, assumption B and the conditions required in proposition 1 (in the main body of the paper) hold, then the only equilibrium outcomes possible are a separating equilibria where the G worker forms his own firm in period 1 and the B worker's strategy is to accept a contract in period 1, or an equilibrium in which both type worker's accept a contract in period one (as described in claim 2). Suppose there is an equilibrium in which the worker types pool on accepting a contract. If a worker deviates and forms his own firm in period one then the belief about that worker will be close to 1 (ratio of mistake probabilities is $\frac{1}{\varepsilon}$ and $\varepsilon \rightarrow 0$). This is because only the G worker can get positive payoffs from forming his own firm. Now, this deviation becomes profitable to the G worker as his payoffs become approximately $2V - R_w$ and therefore we can eliminate equilibria of this kind.

Assumption B is used to restrict beliefs about a worker who makes the costly investment and forms his own firm. Combined with $V < \frac{R_w}{1+\lambda_b(2-\lambda_b)}$, this assumption implies that since only the G type worker can possibly gain by forming his own firm, the beliefs about the worker who plays L in period 1 must be close to 1. This assumption is most natural when the difference between the expected future payoff of a G and B worker is large. If this difference is small, then this assumption becomes a strong one. The difference is inversely proportional to λ_b . Therefore, assumption B is mild if used when the difference in abilities of the two types are large. The next proposition formally describes the conditions needed to obtain the separating equilibrium as the the unique equilibrium.

Proposition 1. *Let Assumption A and B hold. Let $V < \frac{R_w}{1+\lambda_b(2-\lambda_b)}$. If $p_g < \frac{R_p - \lambda_b R_w + (2\lambda_b V - R_p)}{R_w - \lambda_b R_w + (2\lambda_b V - R_p)}$, there exists a separating equilibrium in which the G worker leaves to form his own firm in period one and the B worker accepts a zero wage contract in period 1. Moreover, this is the unique equilibrium outcome of the game.*

Proof. $V < \frac{R_w}{1+\lambda_b(2-\lambda_b)}$ guarantees that the only equilibrium outcomes possible are those in which the G worker leaves and the B worker accepts a contract in period one or those in which both type workers accept a contract in period one. The above discussion eliminates the latter. □